

# **Envelopment Methodology to Measure and Compare Subcontractor Productivity at the Firm Level**

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## **ABSTRACT**

This paper describes a conceptual approach to measure and compare productivity of resource utilization at the firm level, adapting a set of techniques known as Data Envelopment Analysis (DEA). Within this approach, the paper addresses the issues of multiple inputs and multiple outputs of a construction firm, level of detail for data collection, and the required transformations to correct for differences among projects. In particular, we focus on the resource management of subcontractors. Subcontractors manage multiple, concurrent projects and must allocate limited resources across these projects. Interaction between projects and resource allocation creates non-linear effects, and therefore the productivity of the firm is not simply the productivity of its projects. The proposed measurement methodology will allow assessment of the impact of different management policies (including many of those proposed by lean construction researchers) on firm performance. It is hoped that this novel approach to productivity measurement will help subcontractors identify efficient practices and superior management policies, and will promote adoption of these policies.

## **KEY WORDS**

Construction productivity, construction efficiency, subcontractor resource management, total factor productivity, data envelopment analysis, lean construction.

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## INTRODUCTION

Construction subcontractors face unique production challenges. Not only must they face a changing set of production problems on individual projects, they need to manage their finite resources simultaneously across many projects. That each of these projects has changing demand for resources in both quantity and time makes the job of managing subcontractor operations extremely difficult. Managers must take a multi-project perspective, seeking to optimize use of resources at the firm level. This is not simply a matter of optimizing activities on individual projects; the discretion of managers to reallocate resources among projects creates non-linear effects and, hence, production decisions must be considered at the firm level (O'Brien 1998, O'Brien et al. 1995). Unfortunately, construction research has all but ignored the firm or multi-project perspective when developing models to aid decision makers. Similarly, models used by practitioners (e.g., project costing methods) stem from a single-project perspective and do not directly support subcontractor resource allocation decisions across projects. Such decisions are made heuristically, and we lack formal models to guide practitioners (O'Brien 2000).

Concurrent with a lack of models, construction research lacks a measurement method to compare the performance of subcontractors at the firm level. Researchers have a long tradition of measuring productivity at the industry or macro-economic level, typically making a longitudinal study of productivity trends (e.g., Bon and Pietroforte 1990), but this high-level analysis does not provide an indication of firm level performance. At the micro-level, there is a vast literature studying productivity at the level of individual projects and project activities (e.g., Goodrum et al. 2000; Thomas and Yiakoumis 1987). This literature studies various influences on productivity both longitudinally and as cross-section studies among contemporaneous projects and/or activities. Man-hours employed and work produced are measured and compared to the past or compared with other firms to obtain measurements of how efficient a firm is in its activities.

These are unsatisfactory measures of firm level performance as they do not incorporate all inputs and all outputs. There are "total factor" measures of productivity that relate the performance of all inputs to all outputs. There are also "partial factor" measures of productivity that typically relate a single input to output (Link 1987). The scope of most construction productivity research has been to focus on partial measures, principally labor. While useful at the activity level, partial factor metrics are limited. In particular, they do not address complex interactions between different factors both within and across the projects a firm is working on. Thomas and Yiakoumis' (1987) propose a (total) Factor Model to measurement of productivity on activities. They suggest that several influences on productivity (e.g., skill level, weather, site conditions, management, etc.) are separable and additive in nature. Their model does not address interactions within and across projects and hence is not applicable at the firm level. However, their work can be viewed as an important pre-cursor to the proposed methodology in this paper.

This lack of a firm level productivity measurement methodology for subcontractors is a serious gap in construction research that likely retards industrial adoption of new methods. We claim that a measure of productivity at the firm level has a host of benefits, as it:

- Supports subcontractors' management decisions about resource utilization across projects for the most return.
- Supports decisions about investment in resources and in mix of projects.
- Supports benchmarking, allowing subcontractors to better understand their competitive position and improve their performance.
- Supports comparative research of various management policies.

In particular, a firm level productivity measurement methodology allows empirical evaluation of improved management policies promoted by lean construction researchers. For example, Last Planner and production shielding (Ballard and Howell 1998) are productivity improvement techniques built on the project and activity level. While these techniques have proven useful, there is no methodology that can relate the activity and project level performance to firm performance. The methodology envisioned in this paper would allow such an evaluation, allowing comparison of, for example, different variations of Last Planner implementations in sheet metal subcontractors.

A strong motivation for the need for a reliable firm-level productivity metric comes from the International Motor Vehicle Program (IMVP) study that resulted in the famous book *The Machine That Changed the World* (Womack et al. 1990). This book documented the productivity improvements that came with lean production. These documented improvements were a major catalyst for the widespread adoption of those techniques by the automobile industry. Krafcik (1988) developed the assembly plant productivity measurement methods for the study, guided by “the need for the development of methods which would permit accurate assessments of assembly plant performance and aid the identification of superior manufacturing practices in the automotive industry.” The study collected data from 60 assembly plants in 15 countries, representing almost a third of world automotive assembly capacity. The IMVP study is useful as a practical example of the methodological difficulties involved in comparing performance at the firm level, in particular the need for an “apples-to-apples” comparison of technologies. Krafcik addressed this problem using a series of conversion factors, scaling the output of each plant to a standard compact car.

Unfortunately, subcontractor production is not so easily converted to a single standard output. Although specialist firms, subcontractors produce a range of outputs on unique projects, with influencing factors harder to control than a factory environment. Similarly, the range of inputs to subcontractor production is on the same order of magnitude as that of an assembly plant. Thus a productivity measurement methodology to evaluate subcontractor performance at the firm level needs to be significantly more robust than the methods employed in the IMVP study. We propose in this paper that a set of non-parametric, frontier evaluation methods known as Data Envelopment Analysis (DEA) is sufficiently powerful to accommodate the measurement challenge posed by subcontractor production. In the following sections, we describe DEA, its applicability to measuring subcontractor productivity at the firm level, and several research issues that must be addressed to fully adapt DEA as a construction measurement tool.

## FRONTIER PRODUCTIVITY AND DATA ENVELOPMENT ANALYSIS

The IMVP study is a benchmarking study and can be considered a form of frontier analysis in that it identified best practices (i.e., the frontier) and measured other performance relative to best practice. Farrell (1957) in his pioneering work on productive efficiency through frontier analysis, proposed the notion of the structural efficiency of an industry. Structural efficiency is essentially an indication of the dispersion of overall efficiency among the constituent firms in an industry. It measures the extent to which an industry keeps up with the performance of its own most efficient firms. The Farrell approach utilizes the classic econometric production function as its measurement base and estimates the relative level of a firm's efficiency by where it is positioned within the production "frontier." This approach enables firms to assess their relative efficiencies vis-à-vis other firms in the industry. Farrell's work and subsequent development provides a rich theoretical and methodological basis from which to develop measures of firm level performance able to address the difficulties posed by subcontractor production.

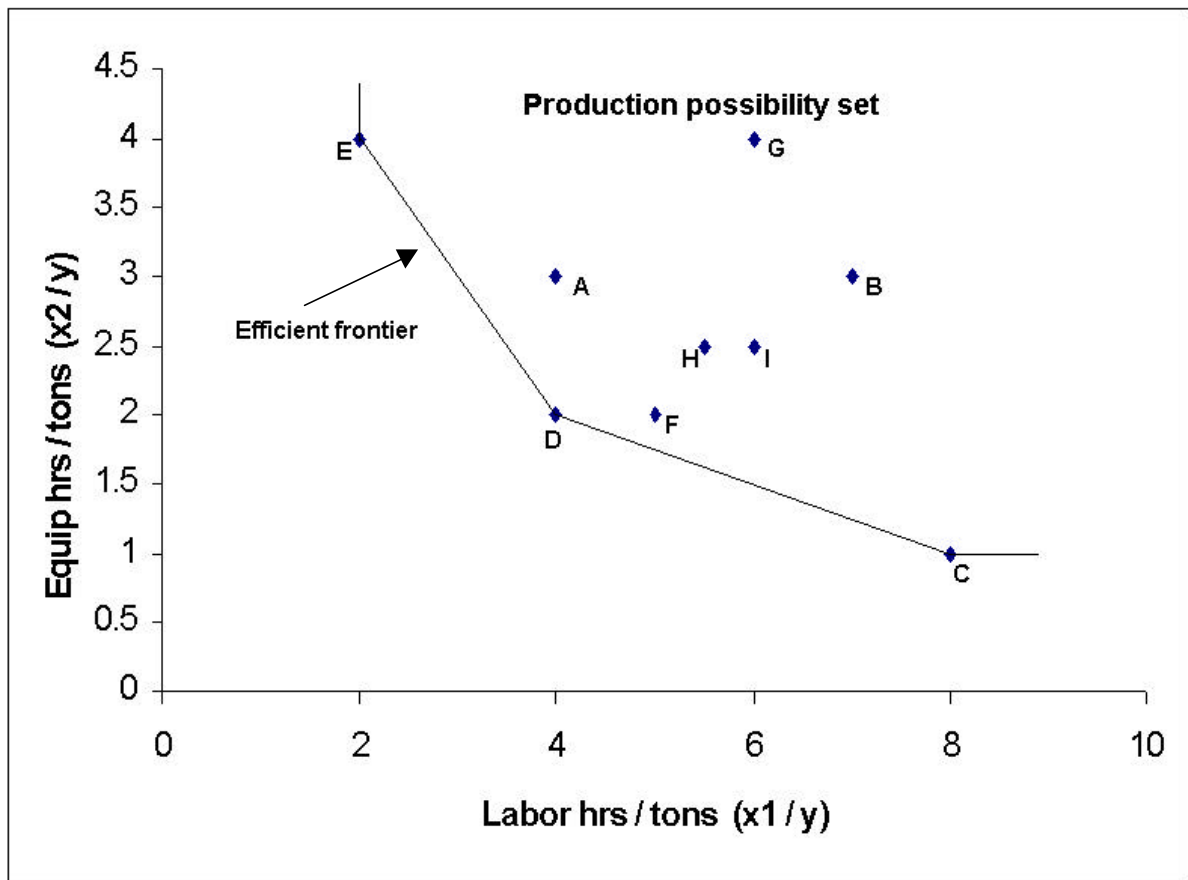
In particular, we believe a generalization of Farrell's framework by Charnes, Cooper, and Rhodes (1978) can be adapted for use in construction. The Charnes, Cooper, and Rhodes (CCR) model reformulated Farrell's model as a mathematical programming approach that can accommodate multiple outputs. The CCR approach initiated development of a broader set of non-parametric, mathematical programming efficiency measurement methods collectively known as Data Envelopment Analysis (DEA). DEA is concerned with evaluations of performance and it is especially concerned with evaluating the activities of organizations such as business firms, hospitals, government agencies, etc. In DEA, the organization under study is called a DMU (Decision Making Unit). A DMU is regarded as the entity responsible for converting inputs into outputs and whose performance is to be evaluated. DEA utilizes mathematical linear programming to determine which of the set of DMUs under study form an envelopment surface. This envelopment surface is referred to as the empirical production function or the efficient frontier. DEA provides a comprehensive analysis of relative efficiency for multiple input-multiple output situations by evaluating each DMU and measuring its performance relative to this envelopment surface. Units that lie on (determine) the surface are deemed efficient in DEA terminology. Units that do not lie on the surface are termed inefficient and the analysis provides a measure of their relative efficiency.

For illustration, we provide the following simple example. Table 1 lists the performance of 9 steel subcontractors each with two inputs and one output. Input x1 is the number of labor hours, Input x2 is the number of welding machine hours. Output y represents tons of open web joists installed in place.

**Table 1: Steel subcontractors example 1**

Subcontractor	A	B	C	D	E	F	G	H	I
Labor hrs (x1)	4	7	8	4	2	5	6	5.5	6
Equip. hrs (x2)	3	3	1	2	4	2	4	2.5	2.5
Open web joists in tons (y)	1	1	1	1	1	1	1	1	1

Figure 1 plots the subcontractors Input  $x_1$ /Output  $y$  and Input  $x_2$ /Output  $y$  as axes. From the efficiency point of view, it is natural to judge subcontractors that use fewer inputs to get one unit of output as more efficient. We therefore identify the line connecting C, D, and E as the “efficient frontier.” This frontier should touch at least one point and all points are therefore on or above (in this case) this line. Note that we can “envelop” all the data points within the region enclosed by the frontier line, the horizontal line passing through C and the vertical line through E. The “enveloped” region is called the “Production Possibility Set.” This means that the observed points are assumed to provide empirical evidence that production is possible at the rates specified by the coordinates of any point in the region.

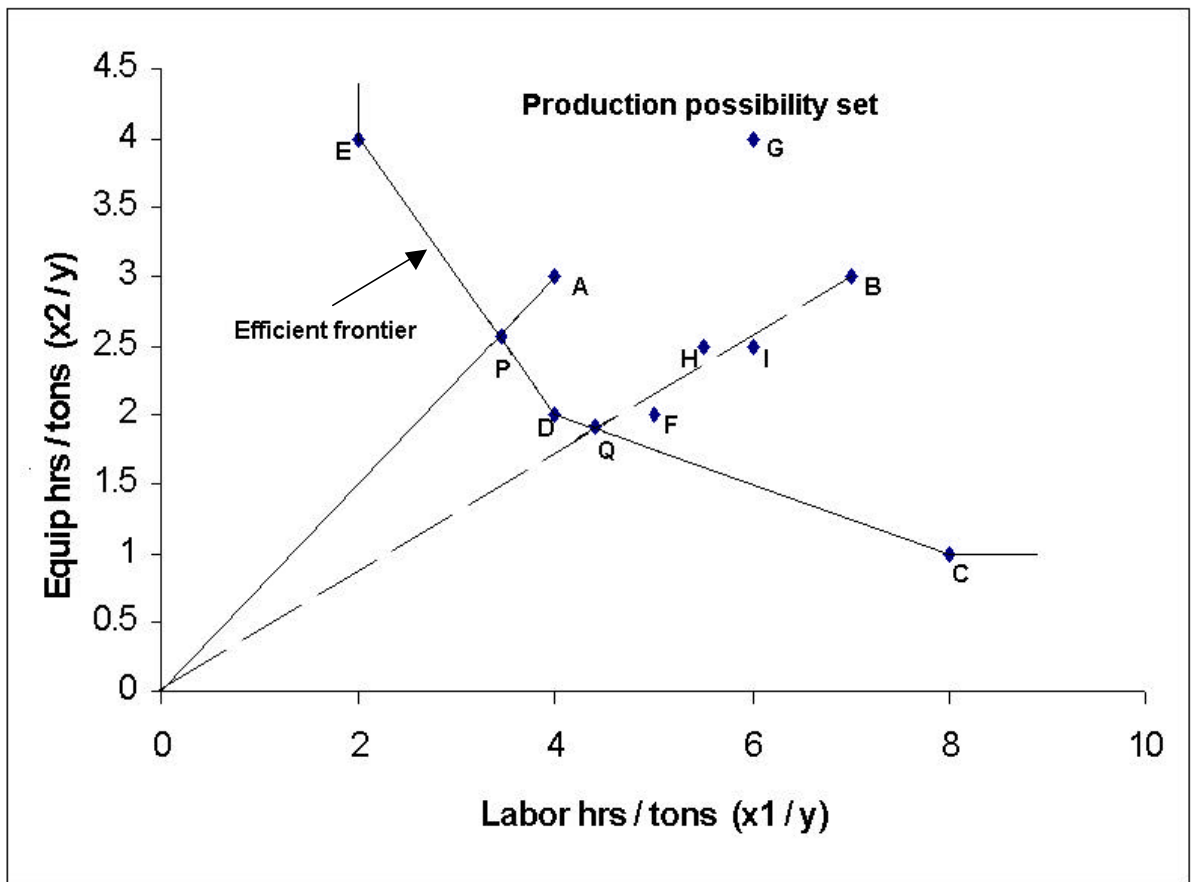


**Figure 1:** Steel subcontractors’ efficiency (example 1)

The efficiency of subcontractors not on the frontier can be measured by referring to the frontier point as follows. For example, subcontractor “A” is inefficient. To measure its inefficiency (see Figure 2), let OA, the line from zero to A, cross the frontier line at P. Then, the efficiency of A is to be evaluated by :  $OP/OA = 0.8571$ . This means that the inefficiency of A is to be evaluated by a combination of D and E because the point P is on the line connecting these two points. D and E are called the “reference set” for A. The reference set

for an inefficient subcontractor may differ from one to another. For example, B has the reference set composed of C and D in Figure 2.

Now we extend our analysis to identify improvements by referring inefficient behaviors to the efficient frontier. From Figure 2, subcontractor A for example, can be effectively improved by movement to P with Input  $x_1 = 3.4$  and Input  $x_2 = 2.6$ . More broadly, the firm can improve its efficiency by adjusting its input mix towards its reference set (D and E in this example). In the same sense subcontractor B can be improved by movement to Q with Input  $x_1 = 4.4$  and Input  $x_2 = 1.9$ .

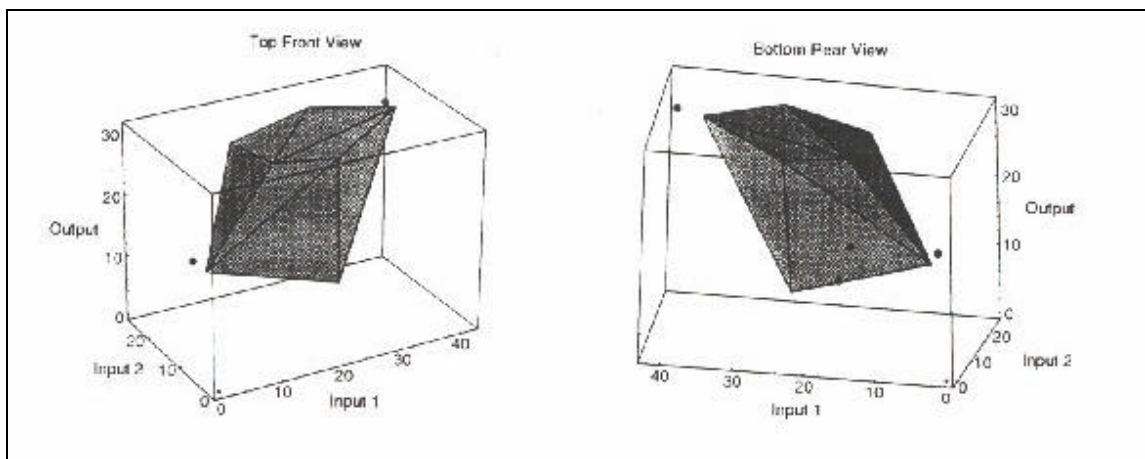


**Figure 2:** Efficiency of subcontractors “A” and “B”

### DEA IN MORE COMPLEX APPLICATIONS

Our example above is simple and does not capture the complexities of the construction environment. However, DEA capabilities and successful deployment in other industries suggest that it is well suited to address the complexity of measuring construction subcontractor productivity. Cooper et al. (2000) argue that DEA has opened up possibilities

for use in cases which have been resistant to other approaches because of the complex nature of the relations between the multiple inputs and multiple outputs involved in many of these activities. DEA has also been used to supply insights into benchmarking practices. Cooper et al. provide instances where DEA has identified numerous sources of inefficiency in some of the most profitable firms – firms that served as benchmarks by reference to their profitability criterion. DEA’s empirical, non-parametric approach does not require the foreknowledge of the production function, but rather allows the determination of an upper boundary for that function. The absence of a priori assumptions in DEA provides it with the flexibility to be used in the construction industry. In sharp contrast to parametric approaches, the weights of the several inputs and several outputs are derived directly from the data, and the user is not required to assign any weights for those inputs and outputs. More importantly, DEA can easily incorporate multiple inputs and multiple-outputs. Thus, it allows the consideration of all resources and production of a construction subcontractor. The use of linear programming provides DEA with the capability to handle large numbers of variables and relations (constraints) and this relaxes the requirements that are often encountered when one is limited to choosing only a few inputs and outputs because the techniques employed will otherwise encounter difficulties. This DEA multiple input, multiple output capability is depicted in Figure 3. A top-front view and a bottom-rear view for a three-dimensional envelopment surface are plotted. The envelopment surface consists of hyperplanes that form particular facets of a convex hull. As mentioned earlier, the efficient DMUs form the envelopment surface and the inefficient DMUs lie below this envelopment surface.



**Figure 3:** Three-dimensional envelopment surface (Ali and Seiford 1993, p.123), demonstrating DEA capabilities to scale to multi-dimensional inputs and outputs

DEA models are either input-oriented or output-oriented. For an input-oriented projection, one seeks a projection such that the proportional reduction in inputs is maximized (i.e., by how much can input quantities be proportionally reduced without changing output

quantities?). Similarly, for the output-oriented projection, one seeks a projection such that the proportional augmentation in outputs is maximized (i.e., by how much can output quantities be proportionally expanded without changing input quantities?). Coelli et al. (1998) and Lovell (1993) argue that linear programming does not suffer from such statistical problems as simultaneous equation bias, the choice of an appropriate orientation is not as crucial as it is in econometric estimation. Thus, for example, if producers are required to meet market demands, and if they can freely adjust input usage, then an input-oriented model seems appropriate. Essentially, one should select the orientation according to which quantities (inputs or outputs) the managers have most control over. Figure 1 shows the input-oriented model for the steel subcontractor example 1, while Figure 4 shows the output-oriented model for the same example. In contrast to the input-oriented model, the output-oriented model uses Output/Input1 and Output/Input2 as axes. As a result, the inefficient subcontractors lie below the efficient frontier.

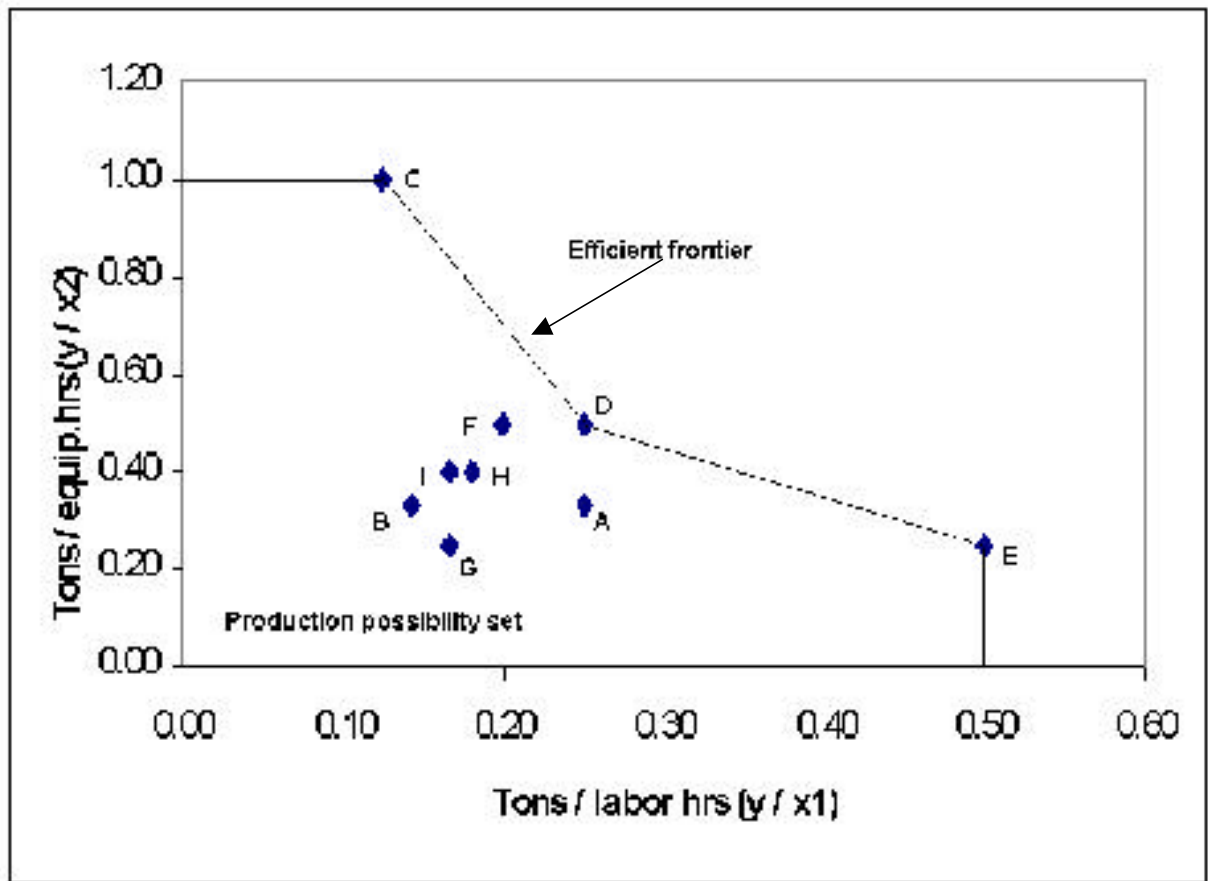


Figure 4: Output-oriented model for the steel subcontractor example 1



Additional features of DEA that makes it plausible for use in the construction industry lie in its ability to accommodate both categorical variables and non-discretionary variables. Frequently, an input or output variable may reflect the presence or the absence of a particular situation (e.g., a subcontractor is working in a certain region in the United States or not). In such situations, one may wish to ensure that a subcontractor is compared with subcontractors that work in the same region under the same weather conditions, and are subjected to the same applicable codes for example. Ali and Seiford (1993) provide another example for the use of categorical variables, where branches of a bank are being compared. Some branches have a drive-in facility, while other branches do not have such a facility. Clearly, it would be a cleaner comparison if the group of DMUs consisted only of branches with a drive-in facility. However, Ali and Seiford (1993) argue that the use of categorical (input or output) variables allows the incorporation of such binary factors and can improve the construction of the efficient frontier.

Non-discretionary variables are variables over which a DMU has no control. In some instances, a subcontractor may not be able to alter some input quantities. That is, these input quantities cannot be varied at the discretion of the subcontractor but nevertheless need to be taken into account in arriving at relative efficiency evaluations. For example, number of non-working days because of weather conditions, etc. Since these variables are not under the control of the subcontractor, it makes no sense to minimize their input quantities. Banker and Morey (1986) utilize the non-discretionary variables in analyzing a 60-DMU network of fast food restaurants. Six inputs are considered: expenditures for supplies and materials, expenditures related to labor, age of store, advertising expenditures allocated to store by headquarters, presence/absence of drive-in window, and location in urban versus rural area. Only the first two inputs are under the control of the individual restaurant manager. Thus, information concerning efficiency gains stemming from reduction in these expenditures would be useful as a management tool. Information about gains from reduction in the non-discretionary inputs is less useful for operations. Fortunately, DEA techniques can address these differences in inputs.

## **METHODOLOGY TO MEASURE AND COMPARE SUBCONTRACTOR PRODUCTIVITY AT THE FIRM LEVEL**

In this section, we present the CCR-model (Charnes, Cooper, and Rhodes, 1978) of DEA to demonstrate some of the technical details involved and to motivate further research. In particular, we focus on the dual of the CCR-model to measure and compare subcontractor productivity at the firm level. Building from the example in section 2, we model construction subcontractors as multiple-input, multiple-output Decision Making Units (DMUs) that attempt to minimize their inputs for given outputs. As such, our model takes an input-oriented rather than an output-oriented approach.

Suppose we have  $n$  DMUs with  $m$  input items and  $s$  output items. Let the input and output data for  $DMU_j$  be  $(x_{1j}, x_{2j}, \dots, x_{mj})$  and  $(y_{1j}, y_{2j}, \dots, y_{sj})$ , respectively. Therefore, the input data matrix  $X$  is an  $(m \times n)$  matrix and the output data matrix is an  $(s \times n)$

matrix. For each DMU, we form the virtual input and output by (yet unknown) weights ( $v_i$ ) and ( $u_r$ ):

$$\begin{aligned} \text{Virtual input} &= v_1 x_1 + v_2 x_2 + \dots + v_m x_m \\ \text{Virtual output} &= u_1 y_1 + u_2 y_2 + \dots + u_s y_s \end{aligned}$$

Given the data, we measure the efficiency of each DMU once and hence, we need  $n$  optimizations, one for each DMU $_j$  to be evaluated. Let the DMU $_j$  to be evaluated on any trial be designated as DMU $^\circ$  where  $\circ$  ranges over 1, 2, .....,  $n$ . We solve the following fractional programming problem to obtain values for the input “weights” ( $v_i$ ) ( $i = 1, \dots, m$ ) and the output “weights” ( $u_r$ ) ( $r = 1, \dots, s$ ) as variables.

(FP $^\circ$ )

$$\begin{aligned} \max \quad & \theta = (u_1 y_{1^\circ} + u_2 y_{2^\circ} + \dots + u_s y_{s^\circ}) / (v_1 x_{1^\circ} + v_2 x_{2^\circ} + \dots + v_m x_{m^\circ}) \\ \text{Subject to} \quad & (u_1 y_{1j} + \dots + u_s y_{sj}) / (v_1 x_{1j} + \dots + v_m x_{mj}) \leq 1 \quad (j = 1, \dots, n) \\ & v_1, v_2, \dots, v_m \geq 0 \\ & u_1, u_2, \dots, u_s \geq 0 \end{aligned}$$

The FP attempts to maximize the objective function  $\theta$ , which is the ratio of “virtual output” to “virtual input.” This maximization is achieved by holding the outputs constant, while minimizing the inputs; a fact that will become clear when we discuss the dual. The constraints of the FP mean that this ratio should not exceed 1 for every DMU. The resulted weights ( $v_i$ ) and ( $u_r$ ) from FP maximize the output to input ratio of DMU $^\circ$ , the DMU being evaluated. By virtue of the constraints, the optimal objective value  $\theta = \theta^*$  is at most 1.

The above fractional program (FP $^\circ$ ) is non-linear. As such, linear programming can not be used to solve it. We therefore replace the (FP $^\circ$ ) with the following linear program (LP $^\circ$ ), which is called the CCR-model:

(LP $^\circ$ )

$$\begin{aligned} \max \quad & \theta = (u_1 y_{1^\circ} + u_2 y_{2^\circ} + \dots + u_s y_{s^\circ}) \\ \text{subject to} \quad & (v_1 x_{1^\circ} + v_2 x_{2^\circ} + \dots + v_m x_{m^\circ}) = 1 \\ & (u_1 y_{1j} + \dots + u_s y_{sj}) \leq (v_1 x_{1j} + \dots + v_m x_{mj}) \quad (j = 1, \dots, n) \\ & v_1, v_2, \dots, v_m \geq 0 \\ & u_1, u_2, \dots, u_s \geq 0 \end{aligned}$$

The objective function of the LP is to maximize  $\theta$ , which reflects the output of DMU $^\circ$ . The input of DMU $^\circ$  was set as a constraint that is equal to 1. The other constraint indicates that the outputs of the rest of the DMUs do not exceed their inputs. Clearly, the optimal value of  $\theta = \theta^* \leq 1$ .

Let us suppose we have an optimal solution of (LP $^\circ$ ) which we represent by ( $\theta^*$ ,  $v^*$ ,  $u^*$ ). We can then identify whether (DMU $^\circ$ ) is CCR-efficient or not as follows:

1. (DMU $^\circ$ ) is CCR-efficient if  $\theta^* = 1$  and there exists at least one optimal ( $v^*$ ,  $u^*$ ) with  $v^* > \mathbf{0}$  and  $u^* > \mathbf{0}$ . This simply means that DMU $^\circ$  is on the efficient frontier. Compared to the rest of the DMUs, DMU $^\circ$  effectively converts its inputs into outputs.
2. Otherwise, DMU $^\circ$  is CCR-inefficient.

It is important to note here that the measures of efficiency presented above for both the (FP $^\circ$ ) and the (LP $^\circ$ ) are “units invariant.” In other words, they are independent of the units of measurement used. Thus, for the steel subcontractors in example 1, one firm can measure the

output in tons and the inputs in working hours while another measures these same output and inputs in linear foot and working days respectively. They will nevertheless obtain the same efficiency value.

In linear programming terminology, every LP has a counterpart that is called the dual. When taking the dual of a given LP, we refer to the given LP as the primal. If the primal is a maximization problem, the dual will be a minimization problem, and vice versa. The importance of the dual lies in its ability to provide additional economic insights. In our case, the dual enables us to determine all input excesses and output shortfalls. Based in the preceding discussion, the CCR-efficiency model was formulated as an LP problem with row vector  $\mathbf{v}$  for inputs and row vector  $\mathbf{u}$  for outputs. Both  $\mathbf{u}$  and  $\mathbf{v}$  are treated as variables in the following primal LP problem, which is presented in vector-matrix notation:

$$\begin{aligned}
 (\text{LP}_o) \quad & \max \mathbf{u} \mathbf{y}_o \\
 \text{subject to} \quad & \mathbf{v} \mathbf{x}_o = 1 \\
 & -\mathbf{v} \mathbf{X} + \mathbf{u} \mathbf{Y} \leq 0 \\
 & \mathbf{v} \geq 0, \mathbf{u} \geq 0
 \end{aligned}$$

The dual problem of  $(\text{LP}_o)$  is expressed with a real variable  $\theta$  and a nonnegative vector  $\mathbf{l} = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$  of variables as follows:

$$\begin{aligned}
 (\text{DLP}_o) \quad & \min \theta \\
 \text{subject to} \quad & \theta \mathbf{x}_o - \mathbf{X} \mathbf{l} \geq 0 \\
 & \mathbf{Y} \mathbf{l} \geq \mathbf{y}_o \\
 & \mathbf{l} \geq 0
 \end{aligned}$$

Table 2 shows correspondences between the primal  $(\text{LP}_o)$  and the dual  $(\text{DLP}_o)$ .

**Table 2: Primal and dual correspondences**

Source: Cooper, et al. (2000, p.44)

Constraint ( $\text{LP}_o$ )	Dual variable ( $\text{DLP}_o$ )	Constraint ( $\text{DLP}_o$ )	Primal variable ( $\text{LP}_o$ )
$\mathbf{v} \mathbf{x}_o = 1$	$\theta$	$\theta \mathbf{x}_o - \mathbf{X} \mathbf{l} \geq 0$	$\mathbf{v} \geq 0$
$-\mathbf{v} \mathbf{X} + \mathbf{u} \mathbf{Y} \leq 0$	$\mathbf{l} \geq 0$	$\mathbf{Y} \mathbf{l} \geq \mathbf{y}_o$	$\mathbf{u} \geq 0$

$(\text{DLP}_o)$  has a feasible solution  $\theta = 1, \lambda_o = 1, \lambda_j = 0 (j \neq o)$ . Hence the optimal  $\theta$  denoted by  $\theta^*$ , is not greater than 1. To convert the above inequalities into equalities, we introduce the input excesses  $\mathbf{s}^-$  and the output shortfalls  $\mathbf{s}^+$  and define them as “slack” vectors.

$$\begin{aligned}
 (\text{DLP}_o) \quad & \min \theta \\
 \text{subject to} \quad & \theta \mathbf{x}_o - \mathbf{X} \mathbf{l} - \mathbf{s}^- = 0 \\
 & \mathbf{Y} \mathbf{l} - \mathbf{s}^+ = \mathbf{y}_o \\
 & \mathbf{l} \geq 0, \mathbf{s}^- \geq 0, \mathbf{s}^+ \geq 0
 \end{aligned}$$

To discover the possible input excesses and output shortfalls, we solve the following two-phase LP problem:

$$\begin{aligned}
 (\text{DLP}_o) \quad & \\
 \text{Phase 1} \quad & \min \theta \\
 \text{Phase 2} \quad & \min -\mathbf{s}^- - \mathbf{s}^+
 \end{aligned}$$

subject to

$$\begin{aligned} \theta \mathbf{x}_o - X \mathbf{l} - \mathbf{s}^- &= \mathbf{0} \\ Y \mathbf{l} + \mathbf{s}^+ &= \mathbf{y}_o \\ \theta \geq 0, \mathbf{l} \geq 0, \mathbf{s}^- \geq 0, \mathbf{s}^+ \geq 0 \end{aligned}$$

The objective of phase 2 is to find a solution that maximizes the sum of input excesses and output shortfalls while keeping  $\theta = \theta^*$ . An optimal solution  $(\theta^*, \mathbf{s}^-, \mathbf{s}^+)$  of phase 2 is called the max-slack solution. If the max-slack solution satisfies  $\mathbf{s}^- = \mathbf{0}$  and  $\mathbf{s}^+ = \mathbf{0}$ , then it is called zero-slack. If an optimal solution  $(\theta^*, \mathbf{l}^*, \mathbf{s}^-, \mathbf{s}^+)$  of the two LPs above satisfies  $\theta^* = 1$ , and is zero-slack ( $\mathbf{s}^- = \mathbf{0}, \mathbf{s}^+ = \mathbf{0}$ ), then the DMU<sub>o</sub> is called CCR-efficient. Otherwise, the DMU<sub>o</sub> is called CCR-inefficient. For an inefficient DMU<sub>o</sub>, we can use the following CCR projection formulas to calculate the improved input and improved output:

$$\begin{aligned} \text{Improved input} \quad \mathbf{x}_o &= \theta^* \mathbf{x}_o - \mathbf{s}^- \\ \text{Improved output} \quad \mathbf{y}_o &= \mathbf{y}_o + \mathbf{s}^+ \end{aligned}$$

The above two-phase LP problem is our proposed model to measure and compare subcontractor productivity at the firm level. For illustration, we utilize the steel subcontractors example after modifying the inputs of subcontractors F and G and excluding subcontractors H and I as shown in Table 3. The results of this example are shown in Table 4.

**Table 3: Steel subcontractors example 2**

Subcontractor	A	B	C	D	E	F	G
Labor hrs (x1)	4	7	8	4	2	10	3
Equip. hrs (x2)	3	3	1	2	4	1	7
Open web joists in tons (y)	1	1	1	1	1	1	1

**Table 4: Steel subcontractors example 2 results\***

Subcontractor	CCR-Efficiency	Reference set	Excess		Shortfall $s^+$
			$s_1^-$	$s_2^-$	
A	0.8571	D, E	0	0	0
B	0.6316	C, D	0	0	0
C	1.0	C	0	0	0
D	1.0	D	0	0	0
E	1.0	E	0	0	0
F	1.0	C	2	0	0
G	0.6667	E	0	0.6667	0

\* Results were obtained using the DEA-Solver software (Cooper et al., 2000)

In the following discussion, we only explain the results of subcontractors A, B, and F.

**(DLP) for A is:**

$$\begin{aligned} \text{Phase 1} \quad & \min \theta \\ \text{Phase 2} \quad & \min -s_1^- - s_2^- - s^+ \\ \text{Subject to} \quad & 4\theta - 4\lambda_A - 7\lambda_B - 8\lambda_C - 4\lambda_D - 2\lambda_E - 10\lambda_F - 3\lambda_G - s_1^- = 0 \\ & 3\theta - 3\lambda_A - 3\lambda_B - \lambda_C - 2\lambda_D - 4\lambda_E - \lambda_F - 7\lambda_G - s_2^- = 0 \end{aligned}$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G - s^+ = 1$$

all variables are nonnegative

The optimal solution for (DLP)<sub>A</sub> is:

$$\theta^* = 0.8571$$

$$\lambda_D^* = 0.7143, \lambda_E^* = 0.2857, \text{ other } \lambda_j^* = 0$$

$$s_1^- = s_2^- = s^+ = 0$$

$$v_1^* = 0.1429, v_2^* = 0.1429, u_1^* = 0.8571$$

$\theta^* = 0.8571 < 1$ , therefore, subcontractor A is inefficient. Since  $\lambda_D^* > 0$  and  $\lambda_E^* > 0$ , the reference set for A is  $E_A = \{D, E\}$ .

$\lambda_D^* = 0.7143$  and  $\lambda_E^* = 0.2857$  show the proportions contributed by D and E to the point used to evaluate A. As we mentioned earlier, subcontractor A can be brought to the efficient frontier by using the CCR-projection formulas:

Improved input 1 of subcontractor A =  $\theta^* x_1 - s_1^- = 0.8571 (4) - 0 = 3.42$  labor hrs (14.5% reduction).

Improved input 2 of subcontractor A =  $\theta^* x_2 - s_2^- = 0.8571 (3) - 0 = 2.57$  equip. hrs (14.5% reduction).

Improved output =  $y + s^+ = 1 + 0 = 1$  (no change)

The same results are achieved using  $\lambda_D^* = 0.7143$  and  $\lambda_E^* = 0.2857$  as follows:

Improved input 1 of subcontractor A =  $\lambda_D^* \text{ input 1 of D} + \lambda_E^* \text{ input 1 of E}$   
 $= 0.7143 * 4 + 0.2857 * 2 = 3.42$  labor hrs

Improved input 2 of subcontractor A =  $\lambda_D^* \text{ input 2 of D} + \lambda_E^* \text{ input 2 of E}$   
 $= 0.7143 * 2 + 0.2857 * 4 = 2.57$  equip. hrs

Again, we can obtain the same results by utilizing the input weights ( $v_1^* = 0.1429$  and  $v_2^* = 0.1429$ ) and the output weight ( $u_1^* = 0.8571$ ).

$v_1^* x_1 = (0.1429) (4) = 0.58$ , therefore the improved input =  $4 - 0.58 = 3.42$ .

$v_2^* x_2 = (0.1429) (3) = 0.43$ . The improved input =  $3 - 0.43 = 2.57$ .

**(DLP) for B is:**

Phase 1                      min  $\theta$

Phase 2                      min  $-s_1^- - s_2^- - s^+$

Subject to

$$7\theta - 4\lambda_A - 7\lambda_B - 8\lambda_C - 4\lambda_D - 2\lambda_E - 10\lambda_F - 3\lambda_G - s_1^- = 0$$

$$3\theta - 3\lambda_A - 3\lambda_B - \lambda_C - 2\lambda_D - 4\lambda_E - \lambda_F - 7\lambda_G - s_2^- = 0$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G - s^+ = 1$$

all variables are nonnegative

The optimal solution for (DLP)<sub>B</sub> is:

$$\theta^* = 0.6316$$

$$\lambda_C^* = 0.1053, \lambda_D^* = 0.8947, \text{ other } \lambda_j^* = 0$$

$$s_1^{-*} = s_2^{-*} = s^{+*} = 0$$

$\theta^* = 0.6316 < 1$ , therefore, subcontractor A is inefficient. Since  $\lambda_C^* > 0$  and  $\lambda_D^* > 0$ , the reference set for B is  $E_B = \{C, D\}$ .

Improved input 1 of subcontractor B =  $\theta^* x_1 - s_1^{-*} = 0.6316 (7) - 0 = 4.42$  labor hrs (36.8% reduction).

Improved input 2 of subcontractor B =  $\theta^* x_2 - s_2^{-*} = 0.6316 (3) - 0 = 1.89$  equip. hrs (36.8% reduction).

Improved output =  $y + s^{+*} = 1 + 0 = 1$  (no change).

**(DLP) for F is:**

Phase 1  $\min \theta$

Phase 2  $\min -s_1^- - s_2^- - s^+$

Subject to

$$10\theta - 4\lambda_A - 7\lambda_B - 8\lambda_C - 4\lambda_D - 2\lambda_E - 10\lambda_F - 3\lambda_G - s_1^- = 0$$

$$\theta - 3\lambda_A - 3\lambda_B - \lambda_C - 2\lambda_D - 4\lambda_E - \lambda_F - 7\lambda_G - s_2^- = 0$$

$$\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G - s^+ = 1$$

all variables are nonnegative

The optimal solution for (DLP)<sub>F</sub> is:

$$\theta^* = 1$$

$$\lambda_C^* = 1, \text{ other } \lambda_j^* = 0$$

$$s_1^{-*} = 2, s_2^{-*} = s^{+*} = 0$$

$s_1^{-*} = 2$ , therefore subcontractor F is inefficient. Since  $\lambda_C^* > 0$ , the reference set for F is  $E_F = \{C\}$ .

Improved input 1 of subcontractor F =  $\theta^* x_1 - s_1^{-*} = 1.0 (10) - 2 = 8$  labor hrs (20% reduction).

Improved input 2 of subcontractor B =  $\theta^* x_2 - s_2^{-*} = 1.0 (1.0) - 0 = 1.0$  equip. hrs (no change).

Improved output =  $y + s^{+*} = 1 + 0 = 1$  (no change).

From Table 4, subcontractors C, D, and E have  $\theta^* = 1$ , and  $s_1^{-*} = s_2^{-*} = s^{+*} = 0$ . These subcontractors satisfy  $\theta^* = 1$  and the zero-slack ( $s^* = \mathbf{0}$ ,  $s^{+*} = \mathbf{0}$ ) criteria, and are therefore claimed CCR-efficient.

## 5.0 Research Challenges

In the preceding sections, we presented two trivial examples to introduce the basic idea behind the DEA terminology and the CCR-model. However, both examples are far from depicting the complexity faced when measuring productivity at the firm level in the construction industry. Adopting DEA is not straight forward, but rather complicated. The following discussion addresses three issues that should be resolved before DEA can be implemented for firm level subcontractor productivity measurement.

- **Level of detail for data collection**

Construction productivity data can be aggregated at various levels, from sub-activities to activities, activities to work (bid) packages, and work packages to projects. Two questions

arise: (1) At which level should data be collected to facilitate reliable comparison? (2) At which level can data be collected efficiently? This second question is important as it facilitates collection of data across (potentially thousands) of construction firms, thus informing other aspects of the methodology. Further investigation is required to answer the fore mentioned questions before DEA can be successfully applied in construction.

- **Required transformations**

A basic requirement for productivity comparison across projects is consistency of units. Collected data may require simple transformations so that the productivity of construction crews doing a variety of work can be expressed in terms of an equivalent output of a single standard item. (While DEA can accommodate multiple outputs, it is unreasonable to expect that all possible outputs will be included in the analysis, and there must be some consolidation of data.) Thus, the productivity of all crews can be calculated for the same standard item during each time period regardless of the work performed. Likewise, crews from different projects can have their productivity calculated for the standard item. This facilitates comparing productivity of crews across projects because all the productivity values represent installing the standard item of work. Such transformations take the form of conversion factors described by Thomas (2000). The conversion factor shows how much more or less difficult an item is to install compared to the standard item. It remains a research issue to determine appropriate conversion factors that are consistent with the level detail of data collected.

- **Inputs and outputs**

Which inputs and outputs should be accounted for in the CCR-model? As Stigler (1976, p. 213-214) has observed, measured inefficiency may be a reflection of a failure to incorporate the right variables and the right constraints and to specify the right economic objective, of the production unit. However, we believe a starting point would be to breakdown the inputs of a construction subcontractor (DMU) into three managerial policies:

- Equipment policies: equipment hours, sum of depreciation of capital equipment owned and expenses on capital equipment leasing, average maintenance expense as a percentage of equipment book value.
- Workforce policies: labor hours.
- Technical staff policies: expenditure on technical staff (salaries, training, etc.)

As for the outputs, we consider each type of work performed by a subcontractor to be an output of that subcontractor. In other words, we treat the physical quantities installed in place as outputs (i.e. SF of concrete masonry units, CF of mortar, etc.). One of the benefits of DEA is its ability to flexibly incorporate different data inputs and output. Unfortunately, with flexibility comes choice, and it remains a research issue to determine which inputs and outputs will be used.

## **6.0 CONCLUSIONS**

This paper presents a Data Envelopment Analysis methodology as an approach to measuring and comparing subcontractor productivity at the firm level. DEA is an empirical, non-parametric approach to productivity measurement that can be extended to multiple inputs (resources) and multiple outputs (products). It is specifically designed to compare

productivity between firms (decision making units or DMUs), ranking them against a frontier defined by the most productive firm(s). DEA appears well suited to measuring the productivity of construction subcontractors. The multi-input capabilities of DEA allow comparison of firms' efficiency employing all their resources (a short-coming of much activity level construction productivity research which focuses only on labor). The multi-output capabilities of DEA allow inclusion of the different types of products (e.g., built-up roofing vs. tiles) performed by the subcontractor; this allows comparison at the firm level to determine not just relative efficiency but also policy questions such as 'what is the best mix of projects?' The determination of a frontier provides not just a relative comparison among firms but also an absolute measure that can be used to measure productivity changes over time. DEA appears to have the requisite power and flexibility to be employed in construction; however, further research is needed to allow effective pre-processing of data for analysis using DEA methods.

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