MULTI-PROJECT RESOURCE ALLOCATION: PARAMETRIC MODELS AND MANAGERIAL IMPLICATIONS

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ABSTRACT
Subcontractors have finite resources they must allocate to multiple projects, often in conditions of competing demand and uncertainty about project schedule. Subcontractors will shift resources fluidly between projects to meet demand, seeking to optimize productivity across projects. Choices about resource allocation are perhaps the most important operational decision that subcontractors make. Despite this, construction research has only recently begun to appreciate the multi-project environment of subcontractors, taking instead a view of production in the context of single projects. As a starting point for a multi-project model, this paper presents a parametric model of subcontractor productivity on a work package. The model relates site conditions, resource allocation, and productivity, allowing quantitative assessment of the impact of shifting resources to or from the work package. An application of the model is presented for one subcontractor with calibrated parametric functions. Use of the model for multi-project resource allocation decision is discussed, and several implications for subcontractor and site management are developed. Many of the central implications are derived from the shape of the productivity modifying functions in work area and resource balance, suggesting a natural categorization of subcontractor technologies.

KEY WORDS
Subcontractor production, resource allocation, multi-project coordination, supply chain management.

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INTRODUCTION

Construction research – both lean and traditional – has generally taken a project centric perspective when modeling production. Traditional production operations research has generally looked to optimize individual construction activities (Oglesby et al. 1989), and with that optimization improve project performance. Lean construction research has criticized the traditional approach and has generally taken a systems/flow view of production, seeking to reduce waste and improve reliability (Koskela 1999). While the value of the lean approach is well demonstrated, this research has continued to view production from the perspective of a single project.

What is missing from most lean and traditional approaches is an appreciation of the multi-project production environment that construction subcontractors operate within. A subcontractor that operates with competing demand for resources across projects (often with variability in demand due to uncontrollable shifts in project schedules) faces a different environment than a lean assembler with a level demand for production. In such an environment, the use of time, inventory, or capacity buffers may not indicate ‘waste’ in an operational sense but an efficient response to complex demands on production. Additionally, the ability to shift resources between projects is a key component of flexibility; subcontractors, unlike suppliers, have limited ability to buffer production by producing ahead of schedule and respond to changes on projects by reallocating resources.

Construction research has not entirely ignored the multi-project resource allocation need of subcontractors or suppliers. Birrell (1980) was the first to explicitly describe that subcontractors needed to balance resource demand across many projects, and suggested that project schedulers maintain a volume of work available such that subcontractors could maintain a constant crew size on each project. The “constant crew size” paradigm is a simplifying assumption that ignores the dynamic nature of projects and the ability of subcontractors to vary the resource intensity on each project, particularly in response to uncontrollable deviations from plan. Unfortunately, much subsequent work in construction has adopted constant crew size as a paradigm without consideration of the multi-project needs of subcontractors.

This author was the first to consider the implications of project uncertainty and change on the cost and resource allocation of subcontractors and suppliers and the need for consideration of resource constraints when coordinating schedule alternatives (O'Brien et al. 1995). Subsequent empirical work has demonstrated that subcontractors and suppliers routinely reallocate resources among projects on a daily basis (O'Brien and Fischer 1999; O'Brien et al. 1997). Building from Ballard and Howell's (1998) Last Planner/production shielding methodology for a single project, Choo has developed a database system that aids in the tracking and assignment of resources across projects as well as multiple trade coordination on a single project (Choo and Tommelein 2000; Choo et al. 1999). While this system does allow users to manually input resource levels for each work package, it is directed towards maintenance of level crew utilization and does not fully capture the richness of a multi-project environment.

Lacking from the literature are parametric models of the subcontractor resource allocation process. Without such models, we cannot predict behavior, describe costs in a multi-project environment, nor prescribe optimal policies. This paper describes a parametric model relating productivity, site conditions, and resource allocation to a work
package and discusses implications of the model in when considering multi-project resource allocation.

MODELING PROJECT PRODUCTIVITY AND RESOURCE ALLOCATION

RESOURCE ALLOCATION CONSIDERATIONS

Subcontractors frequently rebalance resources in response to changing site conditions and project demand (O'Brien and Fischer 1999). What considerations must be taken into account by these firms when making resource allocation choices? These firms must consider the switching and logistics costs of moving resources between projects, the affect of altering the balance of different classes of resources on a project, and the ability of a project to absorb or loan resources with regard to productivity and completion dates. Payment and relationships with construction managers may also be taken in account.

A parametric model useful to subcontractor management must take the considerations above into account. Specifically, such a model must relate resource allocation to the site conditions on a project and determine productivity and therefore cost. It is important to note that subcontractors do not allocate resources to a project so much as they allocate resources to work areas or work packages within a project (Choo et al. 1999). A productivity model on a work package is the smallest identifiable unit that subcontractors allocate resources and is a thus a building block for larger resource assignment models.

Characteristics relevant to productivity on a work package include the physical work area available, the ability to easily shift resources (some resources such as large pieces of equipment are relatively fixed compared to more flexible resources such as labor). Of course, difference classes of resources complement each other and the affect of shifting the ratio between classes may be captured in a function of resource complementarity.

MODEL FORMULATION

The effects of work area and complementarity on work package productivity can be parametrically modeled by adjusting an ideal productivity rate with [0,1] functions. This approach has been taken by Thabet and Beliveau (1994) in a model of work area on productivity. The model below (equation 1) builds from their work, but significantly extends their model to capture the relation between site conditions, resource allocation, and productivity. Equation 1 is a simple but powerful model that provides both qualitative and quantitative predictions of subcontractor resource allocation behavior on a work package. Equation 1 is the core of a broader model of multi-project resource allocation.

As a model of production rate on a work area, define:

\[ \hat{P} = (a_j^T y)CW \]

where:

\[ \hat{P} \] = actual productivity rate in the work area for all resources applied
\[ a = (a_{1j}, a_{2j}) \] = ideal productivity rate per unit of flexible resource \( y_j \) for method \( j \) (assumes an optimum balance between fixed and flexible resources)
\[ y = (y_1, y_2) \] = units of flexible resources applied to the work area, where:
\[ y_1 \] = class 1 or core resources
\[ y_2 \] = class 2 or external resources
\[ x \equiv \text{units of fixed resources applied to the work area} \]
\[ w \equiv \text{size of work area (in square feet or appropriate units)} \]
\[ j \equiv \text{construction method employed on work area} \]
\[ C = f(y,x,j) \equiv \text{complementarity productivity modifier for ratio of fixed to flexible resources given method } j \]
\[ W = f(y,x,w,j) \equiv \text{work area productivity modifier as a function of resources, size of work area, and method } j \]

and:
\[ C, W \in [0,1] \]
\[ y, x \in \{0, 1, 2, 3, \ldots\} \]
\[ j \in \{1, 2, 3, \ldots\} \]
\[ a, w \in [0, \infty) \]

**Productivity and Complementarity**

The first part of the work package productivity model describes an ideal productivity rate for a construction method, \( j \). The rate per unit of flexible resources, \( a \), multiplied by the number of flexible resources, \( y \), gives the ideal productivity rate. Flexible resources are defined as labor and small tools and equipment that are often used by single workers. As such, flexible resources are primarily a measure of labor applied to a project. One nuance has been introduced based on observations — the use of two classes of flexible resources, core and external. Many subcontractors keep a fixed number of workers on staff; they prefer not to hire extra workers on a temporary basis as these are less productive. I reflect this in the definition of flexible resources as the full time workers are class 1 or core while the temporary workers are class 2 or external. These classes may have different production rates, captured in the vector for production rate, \( a \).

Consideration of fixed resources — typically heavier equipment that serves crews rather than individuals — does not play a role in calculation of the ideal production rate in the model. Values for the vector \( a \) assume that the right balance of fixed resources are available for ideal productivity. Any imbalance of fixed to flexible resources is captured in the productivity modifying function complementarity, \( C \). The shape of this function will generally be concave in the relevant region. If there is an ideal ratio of flexible to fixed resources then \( C \) will return a maximum value of 1. As the ratio varies, the value returned by \( C \) will decrease towards zero. Consider figure 2, which depicts the complementarity function for a single fixed resource \((x=1)\).

![Figure 2: Complementarity modifier as a function of the ratio of flexible over fixed resources.](image-url)
As the number of flexible resources (shown as $\Sigma y$) increases from zero, productivity will rise. Depending on the type of equipment (fixed resource) used there may be a minimum number of workers required for production to take place. In this case, the value of $C$ will be zero until a minimum number of flexible resources are available (shown as the vertical line to the zero axis on the left side of the graph). In other cases, the equipment can serve any number of workers productively up to a maximum, and $C$ will be at or near one until that limit is reached. In both cases, once the fixed resource is fully utilized by the flexible resources, adding additional flexible resources will lower the average productivity per unit of flexible resource. This is shown as a decreasing productivity to the right of the maximum point in figure 2. Note that this part of the curve will be concave by definition.

**WORK AREA**

The work area productivity modifier, $W$, is developed from Thabet & Beliveau’s (1994) model. However, I extend and parameterize this function to incorporate resource allocation choices. $W$ is a function of the size of the work area, $w$, labor and equipment (flexible and fixed resources $y$ and $x$), and method $j$. The basic functional shape is determined by the size of the work area, $w$, as shown in figure 3.

If there is no available work area, there is no possible production and the productivity modifier is zero. As work area, $w$, increases, productivity increases. At some point, there is more than adequate work area and the function levels. This basic shape is modified by the application of fixed and flexible resources and the method. For a given set of fixed resources $x$ and method $j$, the work area productivity function describes a surface as shown in figure 4:

![Figure 3: Basic work area productivity modifier function](image)

![Figure 4: Work area modifier function including flexible resources](image)
As the number of flexible resources ($y$) increases, the space needed for maximum productivity increases. Thus the surface bends away from the $(W,y)$ plane as shown in figure 4. As flexible resources are discrete, the surface will not be continuous (but is shown so to simplify). It is important to note that the distance of the surface from the $(W,y)$ plane will not necessarily increase proportionally with each increase in flexible resources; the bend of the surface is also determined by fixed resources. A machine intensive method (fixed resource $x$) may have significant work area requirements and the addition of a crew member (flexible resource $y$) may do little to affect productivity. On the other extreme, a purely labor intensive method (such as painting) will be very sensitive to additional crew members in a fixed work area, $w$.

The three dimensional surface in $(W,w,y)$ space shown in figure 4 assumes a constant number of fixed resources, $x$, and a set method $j$. For each combination of $x$ and $j$ there exists another surface in $(W,w,y)$ space similar to figure 4. The method used will tend to increase or decrease the distance of the surface from the $(W,y)$ plane as some methods are more sensitive to changes in work area, $w$, than are others. As fixed resources are by definition difficult to shift between projects and the number of methods is relatively small, there will be relatively few instances of the surface shown in figure 4. As flexible resources are easy to shift, the surface shown in the $(W,w,y)$ space is a useful representation as it allows rapid determination of the impact of a change in $y$ or $w$ on overall productivity.

**APPLICATION EXAMPLE: PACIFIC CONTRACTING**

Pacific Contracting is a California based roofing specialist known for its aggressive development of lean production techniques. This example shows parameterized complementarity and work area functions developed in consultation with Pacific’s management. As for any subcontractor, Pacific is subject to diminished productivity if there is not enough work area. For example, figure 5 shows the work area productivity modification function for built-up roofing. (Figures 5-7 were developed in consultation with the management of Pacific Contracting, plotted, and submitted to management for correction. The figures represent management’s understanding of the affects of work area and complementarity and their responses.) Figure 5 assumes a standard size crew and method. There are two notable features about this function: First, from 0 to 2,000 square feet of work area, Pacific Contracting maintains a policy of not working. Productivity below 2,000 sf is too low for Pacific to make any money on the project. At 2,000 sf, productivity is roughly 50% of normal and increases roughly linearly with space until 5,000 sf. At 5,000 sf productivity is maximum and the function levels off as predicted. This brings us to the second notable feature: Above 6,000 sf productivity decreases; if a work area is too large, it can de-motivate the workers and they will be less productive.
Figure 5 indicates why Pacific has acted aggressively to adopt lean techniques: Along the \( w \) (work area) axis, there is only a range of 1,000 sf where productivity is at its optimum. Moreover, although Pacific actively works to maintain workers near maximum productivity, it is also significant to note that its policy not to work on the work area does not go into effect until productivity is half normal. Of course, figure 5 only shows a regular size crew (around seven); some workers could be removed from the job increasing productivity for the remaining crew. Nonetheless the shape of the curve and the position of the policy cutoff indicate that Pacific is frequently subject to site conditions which lower its productivity.

Another constraint on Pacific’s actions comes from the presence of equipment — fixed resources, \( x \) — that it must use to complete its work. Consider the complementarity function for a single PVC machine that Pacific employs, shown in figure 6:

Figure 6: Complementarity function for a single PVC machine

A single PVC machine requires at least four workers to operate and thus the function drops to zero at four. The device is fully productive with five workers and can be paired with four to six workers with little effect on overall productivity. The output of the device can only sustain six workers at near full productivity, and additional workers do not add to the overall output. Thus the function converges monotonically to zero. When using a PVC device, Pacific has very little ability to shift flexible resources to or from the job without a dramatic affect on overall productivity.

Contrast the complementarity function of a PVC machine to that of a kettle for a built-up roof shown in figure 7. The kettle can accommodate up to ten workers before productivity per worker drops off significantly. Unlike the PVC machine, the complementarity function is much flatter and does not drop off rapidly on either side of its peak (~7 workers). Thus a project using a kettle has quite a bit of ability to absorb or loan flexible resources with little penalty to per unit output (total output of course will vary).
Figures 5, 6, and 7 all share a common shape in the relevant region (near peak productivity): They have a maximum point or region which falls off to either side. Although not necessarily smooth, they are generally concave. Both the work area function of figure 5 and complementarity function of figure 6 are highly concave as the functions drop rather rapidly to either side of the maximum. Both functions indicate a high sensitivity to changing conditions. In contrast, the function of figure 7 is less concave and therefore affords the subcontractor more flexibility to shift resources between projects. Thus one general measure of the severity of shifting resources on a work package is the concavity of a subcontractor’s productivity modifying functions.

MULTI-PROJECT RESOURCE ALLOCATION

The productivity function developed above can be used to determine the ability of a work package to absorb or loan resources, and from there, consider resource allocation across projects. From equation 1, it is straightforward to determine the cost of loaning or absorbing resources on a particular work area. If there are additional resources added, equations 1 will allow computation of activity duration (and hence any bonus if there is a reward for early completion). Similarly, if resources are taken away from a work area, productivity will decrease and penalties, if any, may come into effect. (Rewards and penalties can be contractually determined or be considered as soft estimates of the impact of a delay on relationships.) With computation of activity duration, it is easy to assess the direct cost of any change by multiplying the duration by the direct cost of resources applied and adding any penalty.

With an assessment of cost on a work package, other considerations can be included to build a picture of total cost of resource (re)allocation. There may be switching costs for moving resources from site-to-site. These costs are likely to be different for each class of resource; by definition the cost of switching a fixed resource between sites will be considerably greater than the cost of switching a flexible resource. The cost of switching will vary depending on the sites chosen. As sites have different geographical distances from each other and there will be some transport cost involved, there will be a different cost for switching each resource between every pair of sites.

Another consideration is that projects are composed of one or more work packages. Projects may have completion requirements (and penalties) apart from those for each work package. The cost of adding or taking away resources from a project must include an assessment of project duration, cost, and associated penalties beyond those directly attached to work packages.

To summarize these understandings, consider that when shifting resources among sites, the subcontractor will seek to minimize costs across three primary areas:

1. For each work package, using equation 1 to determine productivity, and then duration and cost (including any penalties).
2. For each project, considering the duration of each work area as part of project duration and an assessment of any penalties for missing project delivery.
3. For switching resources between sites, where each resource will have a (potentially) different cost for each shift due to varying transportation costs.

There are several constraints on cost minimization: One constraint is availability of resources. Other meaningful constraints include, one, limitations of materials supply as
acceleration on a work area due to additional resources may consume goods faster than provided for. Two, limitations due to methods that require certain types of trained labor and equipment, implying that not all resources can be shifted among all projects. Learning effects may also comprise significant constraints.

I do not attempt in this section to further formalize these costs into a multi-project resource optimization model. However, with the understandings above, an optimizing approach generates several insights about how a subcontractor will shift resources among projects in reaction to changing site conditions or schedule.

First and simplest, if there are idle resources, it makes sense to deploy these to a project requiring acceleration rather than affect other projects. Unless there are unusual costs of deploying these resources, this will always be an optimal approach. It is also worthwhile to note that a potential resource allocation in response to poor site conditions or a work stoppage is to pull a resource off a project and hold it idle. This may sometimes be an optimal response if costs of redeployment are high or if no other project can productively accommodate the resource.

Second, I note that switching resources is costly. As such, it makes sense to not just minimize the cost of shifting resources once but the cost of all shifting, now and in the future. Thus when shifting resources, a subcontractor should move them to projects in a way that does not cause several subsequent shifts. Consider that adding resources to a work area will generally accelerate its completion (even if productivity per unit decreases). When this work area is completed, the resources must move to other work areas. If the subcontractor completes the work area before another is released to it, those workers will be idle and may need to be shifted to other projects. Therefore, rather than shift resources from one project to one other, an optimal response could be to shift from one project to several so no single project will be accelerated causing a subsequent shift.

Third, should a work package/project need to be accelerated, it makes sense for the subcontractor to draw resources from projects that are ahead of schedule or that can easily be accelerated at a later date to make up for any lost time. In this regard, favorable projects for the subcontractor are those with a large backlog of available work. If a subcontractor is working on a large work area, it may easily spare a few resources for a period and return them later with little impact on overall schedule. Similarly, should a subcontractor have a large work area in a package or several work packages available to be worked on a project, that project can be accelerated at relatively little cost to the subcontractor (at worst, early completion causes one subsequent unplanned shift of resources rather than several). Following Last Planner methods, Pacific Contracting prefers a large backlog on a project so it can better sequence its work and minimize the chance of interference from other trades; here Pacific is considering influences within the work area. A further insight when considering the ability of a subcontractor to shift resources among sites is that a large amount of available work on one site increases a subcontractor’s flexibility. The large site can absorb or loan resources with relatively little impact on cost and with little ramifications for spillover to other sites.

Fourth, a subcontractor with highly concave productivity modifying functions will be more limited in its ability to shift than those with less concave functions. In the application example above, note that the work area modifier for Pacific Contracting had a fairly narrow peak (was highly concave) before productivity dropped off. Adding resources will likely drop the productivity per unit quickly. Thus we may expect a subcontractor with highly concave functions will try to shift resources to several projects rather than concentrating them on a single project. Conversely, a subcontractor with a less concave function has more flexibility in its ability to shift to other projects. As for the
work area modifier, the concavity of the complementarity function between fixed and flexible resources will act to constrain a subcontractor if it is highly concave. Such a complementarity function severely restricts a subcontractor’s actions as adding flexible resources to a project will quickly drop the productivity per unit, while taking flexible resources away may be prohibited due to a strict minimum needed for any production to occur. Thus subcontractors with a highly concave complementarity function will likely shift flexible and fixed resources together as a unit. This increases shifting costs and may cause rapid acceleration of other sites, leading to ripple effects. As such these subcontractors will also be more likely than others to stay on-site in less than productive site conditions.

**CONCLUSIONS**

This paper introduced a parametric model of productivity on a work package relating resources to site conditions. A simple model, it promises quick calibration and implementation among subcontractors. Yet the model provides considerable insight and is a first step towards a broader quantitative model of multi-project resource allocation. Further development of such a model is necessary at both a descriptive and prescriptive level. Subcontractors do routinely reallocate resources, and a multi-project allocation model is necessary to describe this behavior. This will enhance existing models that assume a constant crew size (e.g., (Thabet and Beliveau 1994); it will also enhance those tools that determine construction methods for activities and implicitly assume that resources will be available (e.g., (Fischer and Aalami 1996)). A multi-project resource allocation model is also necessary to prescribe improved behavior. Ballard and Howell (1998) have found a wide range of performance in practice, and a quantitative model may be necessary to incent poor performers to improve. Similarly, a multi-project resource allocation model enhances Ballard and Howell’s Last Planner system; while the value of their methodology has been clearly demonstrated, it is not a multi-project management system that accounts for the uncontrollable variations in projects or conflicting demand for subcontractors’ finite resources.

While a building block of a larger model, the work package productivity model presented above provides considerable insight in how a subcontractor may choose to allocate resources. Summarizing four observations: One, When a site must be accelerated, a subcontractor will put idle resources to work rather than borrowing from existing sites. Two, when a subcontractor shifts resources away from one site, it will likely distribute them to several sites rather than move them to just one other site to avoid over-accelerating a site. Three, a large backlog on a site gives the subcontractor a flexible and low cost way to loan to or absorb from other sites. Four, subcontractors with highly convex work area productivity modifiers must distribute resources to multiple sites (rather than distribute many to a single site) or face a severe productivity penalty. A modification to point four exists for subcontractors with highly concave complementarity functions. While such functions imply that a subcontractor will try to distribute resources to (or from) several sites, the strict pairing between resources needed for any production to occur may require the subcontractor to shift resources as a unit to other sites.

These insights also lead to several practical suggestions for general contractors when negotiating contracts and developing schedules. First, if a problem is anticipated, it may be worthwhile for the contractor to pay subcontractors to hold some of their workers idle (in reserve) if the cost of borrowing from other sites is high. This is like buying insurance for production. Second, provision of a large amount of backlog on a site is valuable to
subcontractors and may be recognized as such monetarily. Third and finally, due to concavity of productivity modifying functions it will be more costly for some contractors to change their resource profiles than it is for others. This can be considered a risk factor when assessing schedules; while these subcontractors are less likely to switch resources, when they do shift they are likely to move a significant amount of resources slowing or stopping production. If general contractors have a choice they may wish to keep such ‘highly concave’ subcontractors off the critical path.

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