

# TOLERANCE MAPPING – PARTITION WALL CASE REVISITED

Colin Milberg<sup>1</sup> and Iris D. Tommelein<sup>2</sup>

## ABSTRACT

Geometric tolerances within a component and assembly combine to form closed loops based on the work-structure, i.e. the geometry, function, construction methods and construction sequence of the component, assembly or system. A tolerance loop is by definition over-constrained. Tolerance loops can cause fit-up or functional problems if the tolerances within the loop are inconsistent. Inconsistency is common because geometric tolerances are not given due consideration in civil systems design and construction.

Milberg and Tommelein (2003) demonstrated how a combination of tolerance mapping and tolerance management techniques from manufacturing research, applied to the case of a simple partition wall, can help designers represent tolerance loops for different system work-structures. This tolerance mapping technique applied to the same case is herein expanded to include a different tolerancing system, representations of the magnitude of each tolerance and further breakdown of the tolerances by direction. The revised and more detailed mapping system is used to illustrate the benefits of the tolerance principles of datum reduction and consistency. The paper shows how the revised mapping system helps illustrate interdependencies within product and process designs and thus develops insights for better work-structuring decisions. The case is a simple one to illustrate the tolerance mapping system and provide a theoretical basis for application to more complex systems.

## KEYWORDS

Tolerances, Constructability, Work Structuring, Lean Construction

---

<sup>1</sup> PhD Candidate, Civil and Envir. Engrg. Department, 215 McLaughlin Hall, Univ. of California, Berkeley, CA 94720-1712, 510/289-2552, FAX 510/643-8919, [cmilberg@ce.berkeley.edu](mailto:cmilberg@ce.berkeley.edu)

<sup>2</sup> Professor, Civil and Envir. Engrg. Department, 215-A McLaughlin Hall, Univ. of California, Berkeley, CA 94720-1712, Phone +1 510/643-8678, FAX 510/643-8919, [tommelein@ce.berkeley.edu](mailto:tommelein@ce.berkeley.edu)

## INTRODUCTION

Milberg and Tommelein (2003) identified a lack of attention by architecture, engineering and construction (AEC) practitioners to the impacts of geometric tolerances on the quality, duration and cost of AEC projects. Tolerances can impact project quality when they are exceeded and accepted because rework is too costly. Tolerances can impact project duration and cost when rework, additional processes or more time consuming and costly processes are required to meet project tolerances. Tolerances are exceeded under the following conditions: when the design tolerance is tighter than the associated process capability, when the accumulation of the individual component tolerances specified exceed the assembly tolerance, when the communication of tolerance constraints are unclear or non-existent, or when there is a lack of care in process execution.

AEC practitioners give little attention to tolerances for several reasons. First is lack of data on geometric variation in AEC processes, i.e. process capabilities. Second is lack of clarity regarding who is responsible for managing tolerances. The tendency is to push the responsibility and blame downstream and rely on skilled labor and rework. Third is current practice of accounting for costs. Practitioners rely on traditional strategies to absorb geometric variations. These strategies are assumed to be necessary and the associated costs are not individually tracked. Thus, there are no means to compare impacts of alternative strategies. Fourth is difficulty in visualizing and describing geometric variations and their accumulation.

In manufacturing, there are standards for tolerance specification and interpretation as well as tools for tolerance analysis and allocation, which evaluate tolerance accumulation and aid in visualization. Adapting these tools for use in civil engineering seems a logical first step for improving tolerance management in AEC. The accumulation of tolerances through an assembly or series of processes is defined by the assembly function. Table 1 shows the design decisions that impact the assembly function and the AEC project phase in which they are typically determined.

Table 1: Assembly Function Decisions and Project Phase (Zhang 1997 and Houten and Kals 1999)

<b>Assembly Function Design Decisions</b>	<b>Project Phase</b>
Feature and Assembly Tolerance Specification	Product Design
System Geometry	Product Design
Datum Priority	Distributed, All Phases
Connection Types	Product or Process Design
Fabrication/ Construction Means and Methods	Process Design, Fab. And Construction
Process Sequence	Process Design and Construction

Work-structuring is the process of breaking work into chunks, assigning them, sequencing them, and defining their handoffs between production units (Ballard et al. 2001). Typical project work-structure would execute product design by a designer, process design by a contractor, fabrication by fabricators and construction by the contractor, in that order. However, this work-structure may not be the most efficient for all projects. Constructability research has shown that decisions regarding construction means and methods may have a

greater influence on project cost, schedule, and quality than product design decisions (CII 1993). The point is that all these decisions are interdependent, in part as a result of the accumulation of tolerances. Also, individual process work-structures are what determine construction means and methods and process sequence. Therefore accumulation of tolerances should be a consideration in the work-structuring of the project both in terms of the major project phases and the work-structuring of processes within a given phase.

Tolerance networks (Tsai and Cutosky 1996) are a tool for graphically representing the path of tolerance accumulation based on a product component breakdown. Milberg and Tommelein (2003) created a new system, tolerance maps, adapted from tolerance networks, to capture the flow of geometric tolerances associated with a given process work-structure. For a given work-structure, the tolerance maps help engineering practitioners to:

- identify tolerance loops or over-constrained sets of tolerances
- identify functional tolerance constraints that may not be directly measured or controlled during manufacturing, construction and inspection
- identify inconsistencies between the specification of tolerances for individual components and assemblies, i.e. tolerance loops
- identify inconsistencies between tolerances and process capabilities
- make comparisons to alternative product designs, process designs and work-structures

Milberg and Tommelein (2003) used tolerance maps to compare different work-structures used for the installation of a partition wall with an embedded outlet box. In this paper the mapping system has been updated to include more work-structure and tolerance information. The paper will describe and discuss the updates, as well as the additional information that can be garnered from the updated maps.

## **CASE REVIEW**

The partition wall example illustrates impacts that tolerances can have on the installation of a standard stud partition wall containing an electrical box for a switch or outlet (Figure 1). This example, selected for its familiarity and simplicity, requires minimal description of the case and allows for the maximum description of the tolerance maps. Tolerances accumulate in this example through variations in the studs themselves, the placement of the studs, the electrical box itself, the placement of the electrical box, the outlet plate itself, the placement of the outlet plate, the drywall sheets themselves, the cutting of the hole in the drywall, the placement of the drywall and the layout of each of these components or features. One problem that can result is illustrated in Figures 2 and 3. Details of tolerances for each component and the assembly of a wall are described in Milberg and Tommelein (2003).

## **GENERAL TOLERANCE PRINCIPLES**

In design, the location and size of various components are specified. Typically it is not the location of components that is important but the relationship between design components or

objects in the field. For example, in the design of a building frame, the location of each column is not as important as the relative distance between them, which determines the load carried by each. The allowable variations in the dimensional relationship between design components are tolerances.

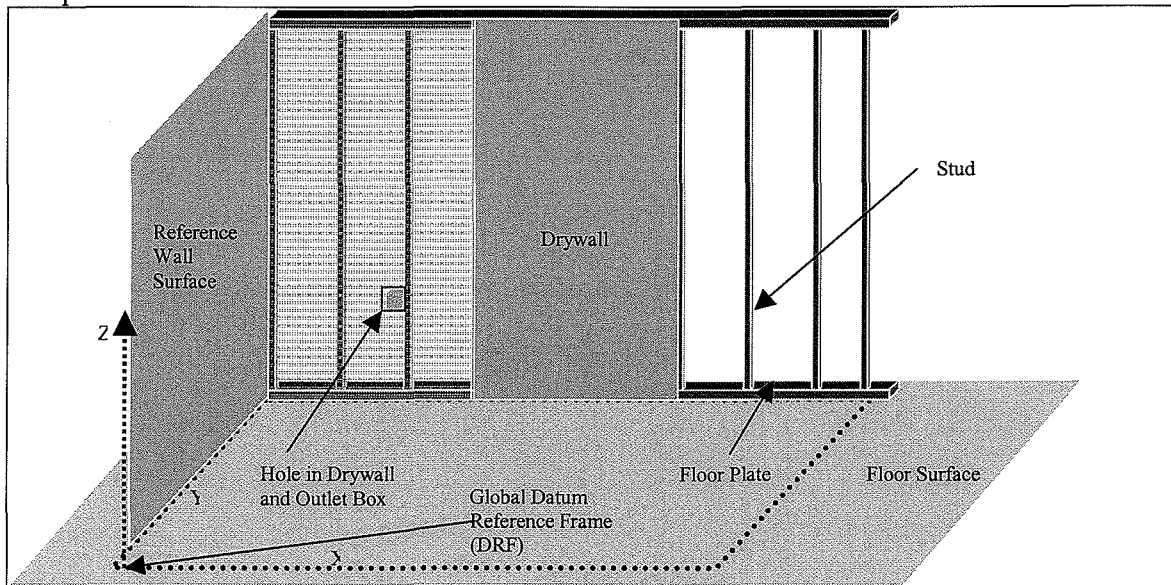


Figure 1: Partition wall with outlet

<p>Figure 2: Hole in the drywall exceeds the limits of the outlet plate</p>	<p>Figure 3: Orientation of Hole in the drywall doesn't match box/outlet orientation</p>	<p>Figure 4: Figure 1 Section View, Looking Down, Taken at the Center of the Electrical Box</p>

Tolerances are referential by definition. For a component feature, geometric and dimensional tolerances (GD&T) are the description of a variation from a nominal geometry. The nominal

geometry is the datum, or theoretically exact reference geometry, such as an axis, plane or straight line. All features within a part or component should be fully constrained, i.e. have uniquely defined locations within a defined reference frame (coordinate system). If a feature is not fully constrained, then it is unnecessary, as it could be located off the part and thus no longer be a feature of the part (Tsai and Cutkosky 1997). Therefore, every feature of a given part will be referenced to the parts datum reference frame (DRF), to another feature within the part, or to another part in the assembly. Similarly, for an assembly, features from one part are referenced to features in other parts or to the assembly DRF in order to form the assembly. Any feature that is used as reference geometry is also called a datum feature. Standards determine the theoretically exact geometry, which replaces the datum feature and is used to define the nominal geometry of an associated feature. The relationships between features consist of two parts: 1) linear and angular dimensions that locate the nominal geometry of the dependent feature from the datum feature's substituted exact geometry; and 2) tolerances (dimensional constraints on variation) relative to that nominal geometry (Henzold 1995).

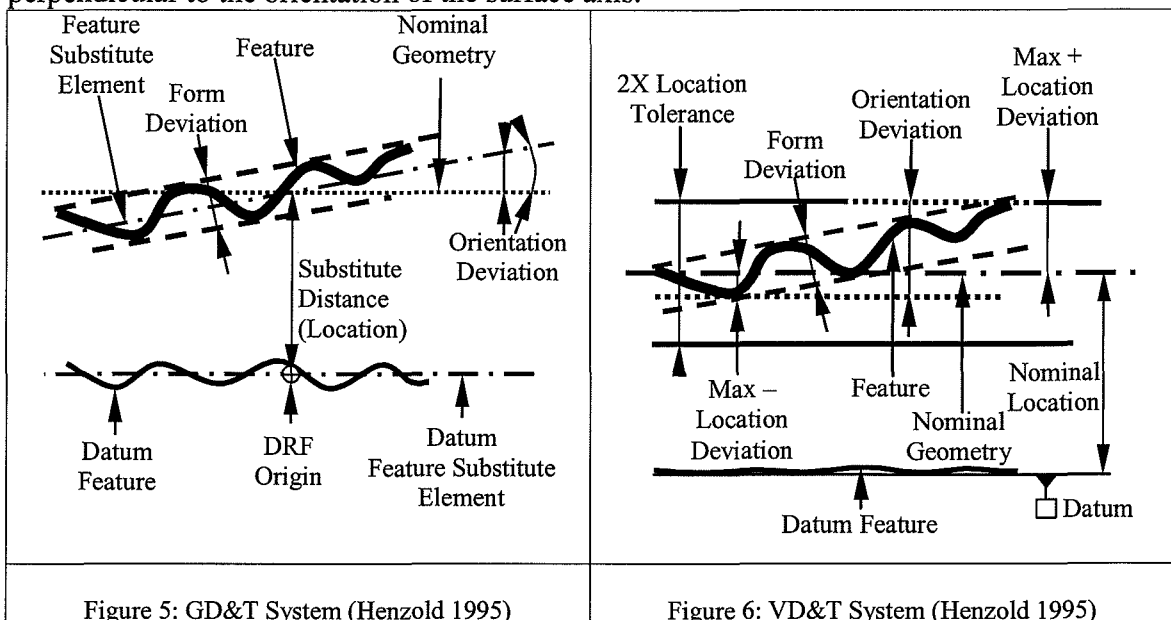
Milberg and Tommelein (2003) described the relationship between size, form, orientation and location tolerances based on the conventional GD&T system (Figure 5). In this paper the vectorial dimensioning and tolerancing (VD&T) system will be used (Figure 6). In the GD&T system form, orientation and location tolerances are hierarchical. Location tolerance zones limit orientation tolerance zone limits, which limit form tolerance zone limits. In the VD&T system, location, orientation and form tolerances are cumulative. The substitute elements in the VD&T system represent the theoretical exact geometry similar to the nominal geometry of the features, in this case lines. The location and orientation of the geometry (best fit line) can be determined by different methods including the Gauss method, which minimizes the sum of squares deviations of the form deviations, and the Chebyshev method, which minimizes the maximum deviations of the form deviations (Henzold 1995). For this paper we have selected the Chebyshev method, as it is similar to the method used in the GD&T system.

In the vectorial system, components have a DRF defined by their primary, secondary and tertiary datum features. In figure 6, the primary datum is the plane defined by the page, call it XY, that defines the Z-axis. The secondary datum is the datum feature substitute element line, which defines the X-axis. This line could also represent the intersection line of a secondary datum plane, XZ, with the primary datum. The y-axis is perpendicular to the X and Z axes. The origin location of the DRF is located by a tertiary datum. In figure 6, the tertiary datum is the right edge of the datum feature (not labeled) and the origin is specified at 2/3 the length of the datum feature substitute element. The origin completes the DRF definition.

The feature nominal geometry and tolerances are specified from the DRF. In the case shown in figure 6, the dotted line represents the feature's nominal geometry. A location vector (the substitute distance in figure 6) describes the location, from the DRF origin, of the origin of the feature's nominal geometry. Location tolerances are described as the maximum deviations from the location vector in each component direction of the DRF. A unit orientation vector at the origin of the nominal geometry describes the orientation of the

nominal geometry. Planes are oriented perpendicular to the orientation vector and lines or axes are oriented parallel to the orientation vector.

In figure 6, the dotted line is also the orientation vector. Orientation tolerances can be specified by linear or angular dimensions. Angular orientation tolerances are described as maximum rotations of the feature's nominal geometry, about the feature's origin, and about each of the DRF axes. Linear orientation tolerances are described as maximum deviations from the orientation vector in each component direction of the DRF. The actual orientation deviation of the nominal geometry about its origin defines the feature's substitute element. The form tolerance is specified as a total distance centered on, and measured perpendicular to, the feature's substitute element. A refinement of the form tolerance, waviness, has a smaller length to depth ratio than the main form tolerance. Waviness is also specified as a total distance perpendicular to, and superimposed over, the form tolerance. Further refinement in specification of roughness can also be made, which is superimposed on the waviness. For circular surfaces (spheres, cylinders, cones and tori), the location and orientation vectors apply to the axes or centers of the surfaces. Both an additional size specification and size tolerance are necessary. Sizes and size tolerances are always perpendicular to the orientation of the surface axis.



## TOLERANCE MAPS

References between features, datum features, DRFs and datum can be represented graphically in the authors' tolerance mapping tool presented in this paper. The tolerance mapping tool is designed to help practitioners apply the general tolerance principles described previously to any given project. The mapping system represents features, datum features and datum as nodes; the connectors between nodes have frames associated with them containing information about the different geometric constraint between the features. The

mapping system graphically represents more product information than found in Tsai and Cutkosky's (1996) tolerance networks and adds work-structuring information as well.

Figure 7 is a key for the different symbols in the authors' tolerance mapping system. The node at the starting end (datum side) of a connector is a datum, DRF, or datum feature. The node at the arrow end (feature side) of a connector is another datum, DRF, or feature. A node can be at the starting end of one or more connectors and at the arrow end of one or more other connectors, which means that the node can be both a feature of a component and a datum feature for another node. When nodes and connectors form a loop with constraints in the same degree of freedom, it indicates that the features in the loop are over-constrained. The loop can be checked for consistency by using numerical tolerance analysis tools and by assigning tolerance values to the connectors between nodes. A loop is consistent if, starting at any node, the worst case accumulation of tolerances through every node does not exceed the allowable tolerance for the connector between the first and last node (Tsai and Cutkosky 1996). Accumulation may also be calculated by statistical means if preferred.

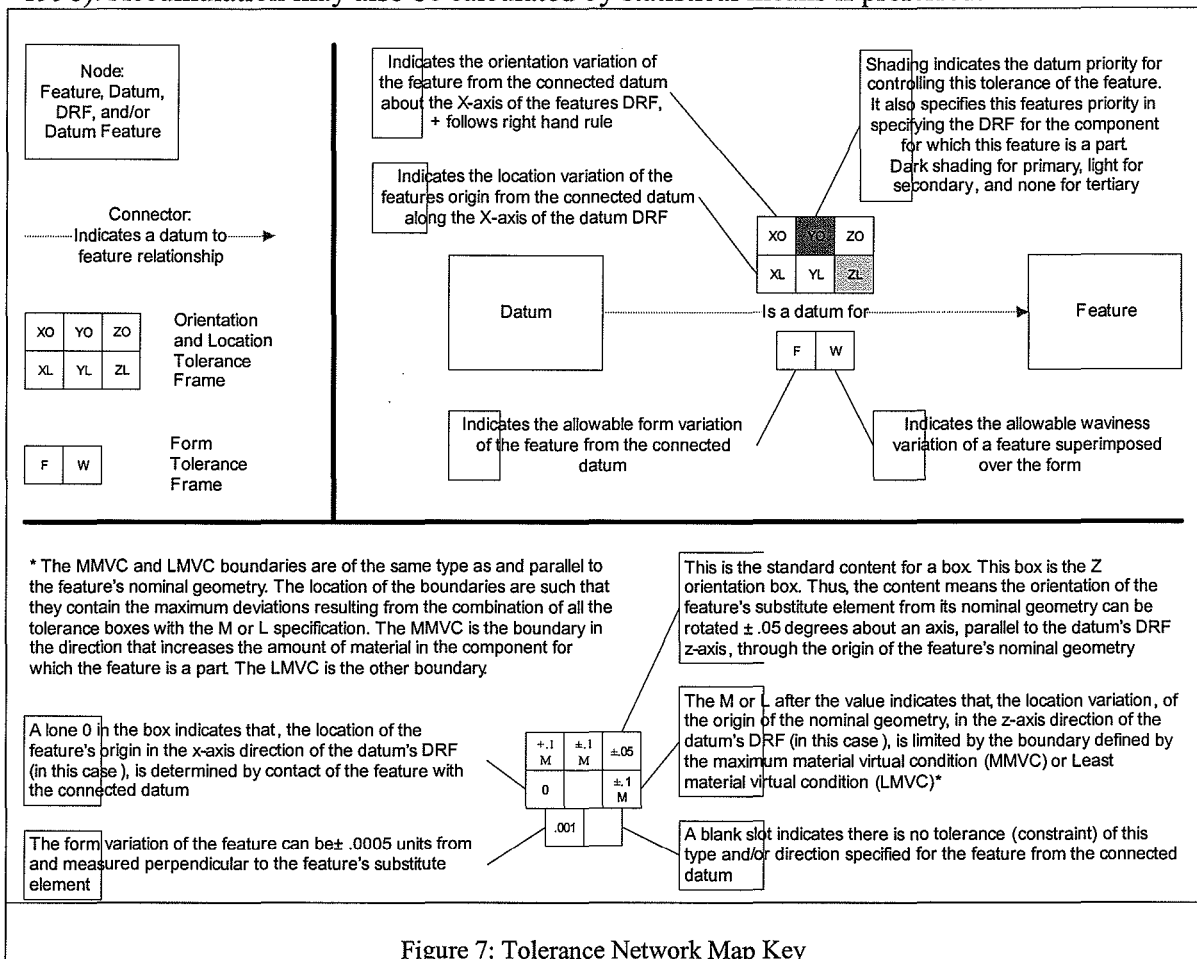


Figure 7: Tolerance Network Map Key

The tolerance values associated with the connectors, i.e. the relationships between nodes, are represented by tolerance frames, which have different boxes for each tolerance variation in each direction (i.e. degree of freedom). The earlier version of the mapping system used

symbols to indicate orientation, location and form variations. The addition of the frame allows further separation by direction (x, y, z) and can include numeric values to indicate the magnitude of the tolerance constraint. The directional information is necessary to identify tolerance loops, and the magnitude information is needed to conduct a tolerance analysis. The separation of the constraints by direction also allows indications for primary, secondary, and tertiary datum, which is important information for relaying design intent. In the key, each box in the frame is labeled to indicate the type of constraint. For example, the box position labeled XO is the place holder for the orientation deviation in the X direction, and ZL is the location deviation in the Z direction. A numerical value in a box represents the magnitude of the tolerance associated with that position in the frame. The value is only the tolerance, i.e. the deviation from the nominal location and orientation, and does not include the location and orientation vectors that define the nominal location and orientation. Although the location and orientation vectors are not shown on the map, they are recorded, as they impact the accumulation of variations among features.

The new version of the mapping system has also added color to the nodes and connectors. Nodes outlined in the same color are part of the same physical component or family of components. The color of the connector indicates the trade or company that controls the tolerances and relationship between nodes defined by the connector and tolerance frames. This way, additional work-structuring information is included in the mapping system.

## **PARTITION WALL TOLERANCE MAP**

Milberg and Tommelein (2003) used an earlier version of their tolerance mapping system to represent process design and sequencing information for the partition wall case. Three alternative sequences (work-structures) were compared. Work-structure 1 was taken from a process perspective, optimizing each task in the sequence and minimizing the number of tasks. The hole in the drywall was positioned relative to the edges of the drywall based on the nominal positions of the edges and the hole. This allowed the holes to be pre-cut away from the work-face in an efficient manner. Work-structure 2 still cut the hole relative to the drywall edges but delayed the cutting until the box was installed. This allowed the actual position of the box to be measured relative to the datum for the drywall edges. The hole could still be cut away from the work-face, from the drywall edges but the cutting is now coupled to the box installation and the extra step of measuring is required. Work-structure 3 involved: putting the drywall in position; pressing it against the box; taking it down and cutting the hole at the marks made by the box. In this case, the cutting is done less efficiently at the work-face and the marking is even less efficient than measuring in terms of resource use. Comparing the maps for the three work-structures for the accumulation of tolerances illustrated the benefits of two tolerance principles: reducing the number of datum in a loop (between work-structure 1 to 2 and 1 to 3); and reducing the magnitude of the tolerances (between work-structure 2 to 3). Work-structure 3 always prevented the problem shown in figure 2 and the associated rework or loss in quality.

The authors apply their new tolerance mapping system described in the previous section to create a map for the partition wall case, shown in part in figure 8. The tolerance map in figure 8 represents the product design of the partition wall. In addition to datum and magnitude reduction, this more in depth mapping system also illustrates another tolerance



management principle: consistency of datum between design (function), manufacturing and inspection (Henzold 1995, Tsai and Cutkosky 1996, Zhang 1997, Houten and Kals 1999).

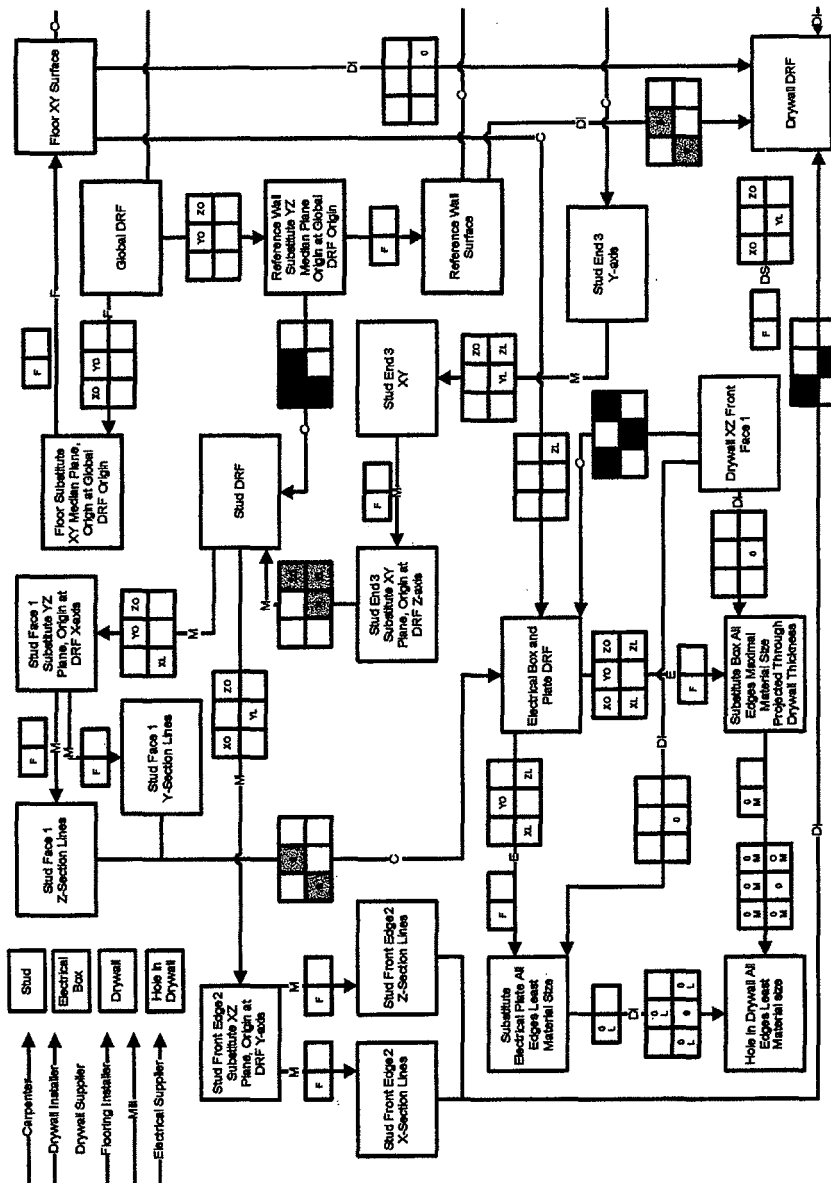


Figure 8: Partition wall with outlet box partial design tolerance network

As the example in figure 8 demonstrates, the edges of the holes in the drywall, which are features of the drywall, are not directly connected to the drywall DRF. When this happens, it means that the trade or company responsible for handling that component (the drywaller in this case) may have to be interrupted by another trade (electricians in this case) before

completing the work associated with that component in order to meet the design tolerances. From a process design perspective, such conditions are undesirable because they increase the number of handoffs and the degree of coupling between activities (Howell et al. 1993). To avoid interruption, one of the trades, say the drywallers, can take as-builts of the other trade's (the electrician's) work (electrical box installation), from which to base completion of their work (cutting the hole in the drywall). This is the condition explained in work-structure 2 (Milberg and Tommelein, 2003), with the disadvantages already noted. If the trade is not interrupted, or as-builts are not taken, then the datum path for the product and process will not match. Inconsistency of the datum between product and process increases the deviations at the closure of the tolerance loop and risks exceeding design tolerances.

Furthermore, the authors' map illustrates that consistency of datum, i.e. a process sequence that is consistent with the design datum sequence, is not even feasible. The hole in the drywall shares the electrical box and plate DRF. The electrical box derives part of its DRF from the drywall DRF. This means that for the process to be consistent with the design datum sequence, the drywall should be installed first, the electrical box and plate should be installed second, and the hole should be cut in the drywall third. Thus, the drywall installation would prevent access to the box installation and without a hole the box will prevent installation of the drywall. Of the three work-structures presented by Milberg and Tommelein (2003), work-structure 3 is the closest to being consistent with the design datum sequence. In work-structure 3, the box is installed first. This is not too much of a deviation, as the drywall only controlled part of the box DRF in the design map. Then the drywall is held in the position of its final installation in every respect but the Y location due to the box. In this way, the hole DRF is that of the electrical box, which is consistent with figure 8. In the other work-structures, the drywall DRF is used for the hole. In this way, the tolerance map identifies locations where the tolerance constraint specified cannot be directly controlled.

## **FURTHER ADVANTAGES AND APPLICATIONS OF TOLERANCE MAPS**

It is also useful to use the authors' mapping system to create process tolerance maps. Process maps would show the sequence of datum to be used to install each component relative to another component previously installed. Comparing a process map to a product map, one would easily identify where a process datum path is not consistent with the product design datum path. In a process map, the boxes in the tolerance frames would contain process capability data instead of design tolerances as in the product map. This way, the values of the process capabilities would be compared easily to the design tolerances.

Ideally, a design tolerance should equal the associated process capability. Designs in which the design tolerance is tighter than the process capability should be avoided as they result in failure to meet the tolerance and potential rework and delays. When the reverse is true, and the process capability is tighter than the design tolerance, there is room to improve the design. One could use a less precise, and thus less expensive, process and/or reallocate the tolerances in order to alleviate tolerances in other locations that are tighter than the associated process capability. Improved communication between designer and contractor or fabricator is facilitated by having both maps, which results in more building solutions that are better integrated and better evaluated.

Currently, however, there is a lack of explicit construction process capability data (Milberg and Tommelein 2003-A). The argument for not collecting this data is that process capabilities vary too much from one job to the next. Yet, without collecting it, how will we know if it varies or what factors cause it to vary? Nevertheless, benefit can be gained even from the rough estimates of process capability.

Another use of the tolerance mapping system is identifying tolerance loops. In figure 8, there are no tolerance loops shown. Although the connectors visually form loops, no feature or component DRF has the same degree of freedom (X, Y, Z - location and orientation) controlled by multiple datum. If for example, the location of the hole in the drywall was also dimensioned and toleranced from the drywall edges, then a loop would be formed. The design would be over-constrained, as the hole's location is controlled both by the drywall DRF and the electrical box DRF.

If a loop is established, then tolerance analysis should be used based on the data in the map. The analysis determines if the total variation in the location of the hole accumulated through the datum sequence, starting at the drywall edges and ending at the electrical box DRF, does not exceed the allowable deviation in location of the hole (relative to the drywall DRF), or vice versa. The loop is consistent, as long as the cumulative variations, starting at a node in the loop and ending in the node connected to the starting node opposite the direction of travel, do not exceed the tolerance associated with the connector between the starting and ending node.

A simpler example of a loop is the dimensioning of columns for a building. If the dimension between each of a set of more than 2 columns is specified with certain tolerances, and the overall dimension from the first and last column is also specified with a certain tolerance, the cumulative tolerance of the dimensions between adjacent columns cannot exceed the tolerance specified between the first and last. Otherwise the system or loop is inconsistent.

The author's mapping system includes more specific information about design intent than is typically found in AEC design drawings and specifications because they use the rules established by the standards for GD&T and VD&T. Current practice in AEC lacks standards for specification of datum and tolerances. This leads to misinterpretation of the designer's intent. Using the column example, if the location of the columns is represented by a series of center to center dimensions with no overall dimension, does this mean that the spacing between studs is more important than the overall dimension from the face of the first to the face of the last column? Many would say yes but there is no AEC standard. Certainly there are recommendations for dimensioning in some codes and specifications but they are only recommendations. In one author's experience, disagreement on the interpretation is frequent. The ambiguity in practice undermines the specification of tolerances, quality control and designer-contractor relations (Birkeland et. al 1971). Tolerance maps avoid these issues and instead: allow clear communication of design intent, specifically avoid unnecessary constraints, and help facilitate concurrent engineering and set-based design.

## **ACKNOWLEDGMENTS**

This research was funded by grant CMS-0116877 from the National Science Foundation, whose support is gratefully acknowledged. Any opinions, findings, conclusions, or

recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation.

## REFERENCES

- Ballard, G., Koskela, L., Howell, G., Zabelle, T. (2001). "Production System Design: Work Structuring Revisited." *White Paper #11*, Lean Constructon Institute, 14 pp. (available at <http://www.leanconstruction.org>).
- Birkeland, P.W. and Westhoff, L.J. (1971). "Dimensional Tolerances in a Tall Concrete Building." *J. or the American Concrete Inst.*, ACI, Detroit, 68 (8) 600-607.
- Bjork, O. (1989) *Computer Aided Tolerancing*. New York: ASME Press.
- CII (1993). *Constructability Implementation Guide*. CII Special Pub. 34-1, Univ. of Texas, Austin, Texas, 277 pp.
- Derucher, K.N. and Korfiatis, G.P. (1988). *Materials for Civil and Highway Engineers*. Prentice Hall, New Jersey.
- Henzold (1995). *Handbook of Geometrical Tolerancing*. Wiley & Sons, England, 413 pp.
- Houten, F. and Kals H. (eds.)(1999). *Global Consistency of Tolerance: Proceedings of the 6<sup>th</sup> CIRP Int. Sem. on Computer-Aided Tolerancing*. Univ. of Twente, Enshede, Netherlands, March. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Howell, G., Laufer, A., and Ballard, G. (1993) "Interaction Between Subcycles: One Key to Improved Methods." *J. Const. Eng. and Mgmt.*, ASCE, NY, 119 (4) 714-728.
- Milberg, C.T., Tommelein, I.D., and Alves, T. (2002). "Improving Design Fitness Through Tolerance Analysis and Allocation." *Proc. Concurrent Engrg. in Construction Conf.*, U.C. Berkeley, CA, 181-193.
- Milberg, C.T and Tommelein, I.D. (2003). "Application of Tolerance Analysis and Allocation in Work Structuring: Partition Wall Case." *Proc. 11th Annual Conference of the International Group for Lean Construction*, 22-24 July, Blacksburg, VA.
- Milberg, C.T. and Tommelein, I.D. (2003-A). "Role of Tolerances and Process Capability Data In Product and Process Design Integration." *Proc. Constr. Research Congress 2003*, Mar. 19-21, Hawaii, ASCE. 8 pp.
- Prasad, B. (1996). *Concurrent Engineering Fundamental, Vol. 1*. NJ: Prentice Hall PTR.
- Reader's Digest Staff (1991) *New Complete Do-it-yourself Manual*. Reader's Digest Association, Inc., Pleasantville, NY, 528 pp.
- Tsai, J., Cutkosky, M.R. (1997). "Representation and Reasoning of Geometric Tolerances in Design." *Artificial Intelligence for Engrg. Design, Analysis and Manufacturing*, Cambridge University Press, 11, 325-341.
- Trabelsi, A. and Delchambre, A. (2000). "Assessment on Tolerance Representation and Tolerance Analysis in Assemblies." *Concurrent Engineering: Research and Applications*. Technomic Publishing Co., Lancaster, PA, Dec., 8 (4) 244-262.
- USG Co. (2003). "Installation and Finish Guide." *Technical Report J371 rev. 2-03*, United States Gypsum Company, Chicago, IL., 26 pp. (available at <http://www.usg.com>)
- Wakita, O.A., Linde, R.M. (1994). *The Professional Practice of Architectural Working Drawings*. Wiley and Sons, Inc., NY.
- Zhang, G. (ed.)(1997). *Advanced Tolerancing Techniques*. Wiley & Sons, NY.